A1

Gegeben sei die Exponentialgleichung:

$$5\cdot\left(2^2\right)^x+\frac{1}{4}\cdot 7^{x+1}=20\cdot 4^{x-2}+\frac{1}{21}\cdot 7^{x+2} \qquad G=\mathbb{R}!$$

Bestimme die Definitionsmenge $\mathbb{D}$ und die Lösungsmenge L.

Es gibt keine Einschränkungen für $\mathbb{D}$ von x, folglich gilt: $\underline{\underline{\mathbb{D}=\mathbb{R}}}$ 0,75

$$5(4)^x+\frac{1}{4}7^x\cdot 7=20\cdot 4^x\cdot\frac{1}{16}+\frac{1}{21}7^x\cdot 49 \quad 0,5$$

$$5\cdot 4^x-\frac{\cancel{20}^{5}}{\cancel{16}_{4}}4^x=\frac{\cancel{49}^{7}}{\cancel{21}_{3}}\cdot 7^x-\frac{7}{4}7^x \quad 0,5$$

$$4^x\left(5-\frac{5}{4}\right)=7^x\left(\frac{7}{3}-\frac{7}{4}\right)$$

$$\frac{4^x}{7^x}=\frac{(\cancel{28}^{7}-21)\cancel{4}^{1}}{\cancel{12}_{3}(\cancel{20-5}_{15})} \quad 0,5$$

$$\left(\frac{4}{7}\right)^x=\frac{7}{45} \quad /\ lg() \quad 0,5$$

$$x\cdot lg\left(\frac{4}{7}\right)=lg\left(\frac{7}{45}\right)$$

$$\underline{\underline{x}}=\frac{lg\left(\frac{7}{45}\right)}{lg\left(\frac{4}{7}\right)}=\underline{3,325} \quad 0,25$$

Kontrolle:

$$502,171+1129,885\overset{?}{=}125,543+1506,513 \quad 0,5$$

$$1632,056=1632,056 \text{ ok.}$$

A2 Bruchungleichung!

a) $$\left.\begin{array}{ll} 2x-1 \neq 0 & x \neq 0{,}5 \\ 0{,}5-x \neq 0 & x \neq 0{,}5 \end{array}\right\} \quad \mathbb{D} = \mathbb{R}\setminus\{0{,}5\}$$

b) $$\frac{3}{2x-1} + \frac{2}{0{,}5-x} + \frac{1}{3} \geq 0$$

$$\frac{3}{2(x-0{,}5)} - \frac{2}{(x-0{,}5)} + \frac{1}{3} \geq 0$$

$$\frac{9-12+2(x-0{,}5)}{6(x-0{,}5)} \geq 0$$

$$\frac{9-12+2x-1}{6(x-0{,}5)} \geq 0 \quad \Rightarrow \quad \frac{2x-4}{6(x-0{,}5)} \geq 0$$

$$\frac{\cancel{2}^{1}(x-2)}{\cancel{6}_{3}(x-0{,}5)} \geq 0 \quad \Rightarrow \quad \frac{x-2}{3(x-0{,}5)} \geq 0$$

oder

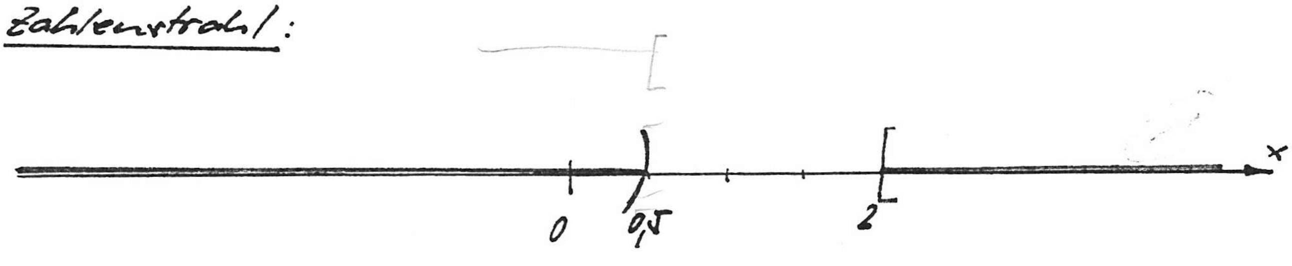
$$\mathbb{L} = \{x \mid x < 0{,}5\,;\ x \geq 2\,;\ x \in \mathbb{R}\}$$

Extrempunkte:

$x = 2 \Rightarrow$ Ungleichung $= 0$!

$x = 0{,}5 \Rightarrow$ Nenner $= 0$!

Zahlenstrahl:



Test mit 0 $\Rightarrow \dfrac{-2}{3(-0{,}5)} \geq 0$ ok.

A3

Für die Logarithmengleichung $\log_4(3x+4)-\log_2(2x+1)=0$ in der Grundmenge der reellen Zahlen ($G = \mathbb{R}$), sind die Definitionsmenge $\mathbb{D}$ und die Lösungsmenge $\mathbb{L}$ zu bestimmen.

$\mathbb{D}$:

$$\left.\begin{array}{lll} (3x+4) > 0 & \Rightarrow & x > -\frac{4}{3} \\ (2x+1) > 0 & \Rightarrow & x > -0{,}5 \end{array}\right\} \quad \mathbb{D} = \mathbb{R}^{>-0{,}5}$$

Lösung:

$$\log_4(3x+4) - \log_2(2x+1) = 0$$

$$\log_4(3x+4) = \log_2(2x+1) \quad \Big| \, 4^{()}$$

$$4^{\log_4(3x+4)} = 4^{\log_2(2x+1)}$$

$$(3x+4) = 2^{2\log_2(2x+1)}$$

$$(3x+4) = 2^{\log_2(2x+1)^2}$$

$$(3x+4) = (2x+1)^2$$

$$3x+4 = 4x^2+4x+1$$

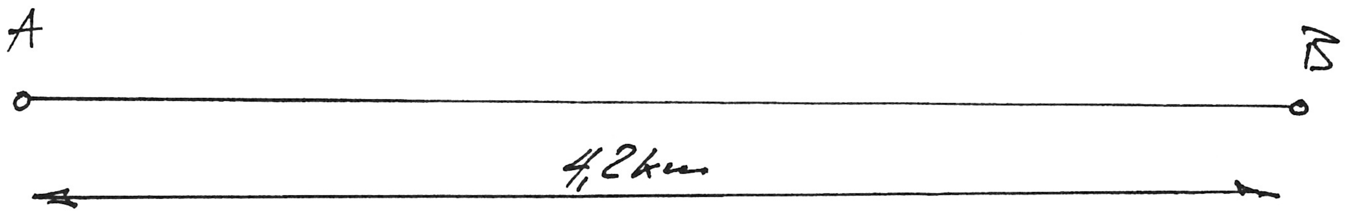
$$\Rightarrow \quad 4x^2 + x - 3 = 0$$

$$x_{1/2} = \frac{-1 \pm \sqrt{1-4\cdot 4\cdot(-3)}}{8} = \frac{-1 \pm 7}{8} \quad \begin{array}{l} x_1 = \frac{-8}{8} = -1 \\ x_2 = \frac{6}{8} = \frac{3}{4} = 0{,}75 \end{array}$$

Kontrolle:
mit $x_2 = 0{,}75$

$$\frac{\log 6{,}25}{\log 4} = \frac{2\cdot\log 2{,}5}{2\cdot\log 2} = \frac{\log 2{,}5}{\log 2} \quad \text{ok} \qquad \mathbb{L} = \{0{,}75\}$$

A4



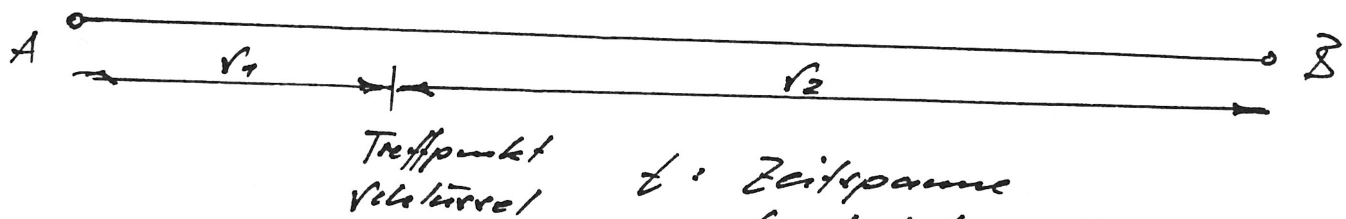
Biker $\underline{v_B} = \frac{s}{t_B} = \frac{4{,}2\,km}{0{,}1\bar{6}\,h} = \underline{25{,}2\,km/h}$ $\quad / \; t_B = 10\,min$

Einrichtzeit : 30 min

Pause : 20 min

Klasse: $t_K = 60\,min \hat{=} 1h$

$\Rightarrow \underline{v_K = 4{,}2\,km/h}$



t : Zeitspanne Sportplatz – Klasse

$$\left.\begin{aligned} s_1 &= v_K\,(t_B + t) \\ s_2 &= v_B \cdot t \end{aligned}\right\} \quad v_K\,(t_B + t) + v_B \cdot t = 4{,}2\,km$$

$$v_K\, t_B + (v_K + v_B)\,t = 4{,}2\,km$$

$$t = \frac{4{,}2 - v_K \cdot t_B}{v_K + v_B}$$

$$\underline{t} = \frac{4{,}2 - 4{,}2 \cdot 0{,}1\bar{6}}{4{,}2 + 25{,}2} = \underline{0{,}119\,h} \quad 7'9''$$

Effektive Pause: $t_p = 20\,min - 2 \cdot 7\,min\ 9\,sec$

$t_p = 20 - 14\,min\ 17\,s$

$\underline{\underline{t_p = 5\,min\ 43\,s}}$

15

Bestimme die gegenseitige Lage beider Geraden zueinander:

$$g: \vec{r} = \begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix} + \mu \cdot \begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix} \quad \text{und} \quad p: \vec{r} = \begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix} + t \cdot \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$$

Falls sich die Geraden schneiden ist der Schnittpunkt und der Zwischenwinkel zu berechnen!

Parallelität

Richtungsvektoren $\vec{a} = \begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix}$

$\vec{b} = \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$

$\vec{a} = \lambda \vec{b}$

$$\begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix} \qquad \left.\begin{matrix} \lambda_1 = 2 \\ \lambda_2 = \in \mathbb{R} \\ \lambda_3 = -\frac{5}{3} \end{matrix}\right\} \text{ g und p sind nicht parallel!}$$

0,5

g ∦ p

Bestimmung von μ_s und t_s:

$$\left| \begin{matrix} 6 + 4\mu_s = 2 + 2t_s \\ 1 = 1 \\ 3 + 5\mu_s = 9 - 3t_s \end{matrix} \right| \Rightarrow \left| \begin{matrix} 4\mu_s - 2t_s = -4 \\ 5\mu_s + 3t_s = 6 \end{matrix} \right|$$

0,5

$$D = \begin{vmatrix} 4 & -2 \\ 5 & 3 \end{vmatrix} = 12 - (-10) = 22$$

$$D_1 = \begin{vmatrix} -4 & -2 \\ 6 & 3 \end{vmatrix} = -12 - (-12) = 0$$

$$D_2 = \begin{vmatrix} 4 & -4 \\ 5 & 6 \end{vmatrix} = 24 - (-20) = 44$$

$$\Rightarrow \mu_s = \frac{D_1}{D} = \frac{0}{22} = 0, \quad t_s = \frac{D_2}{D} = \frac{44}{22} = 2 \quad \text{einsetzen in g oder p}$$

0,5

$$\Rightarrow S\begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix}$$

Zwischenwinkel

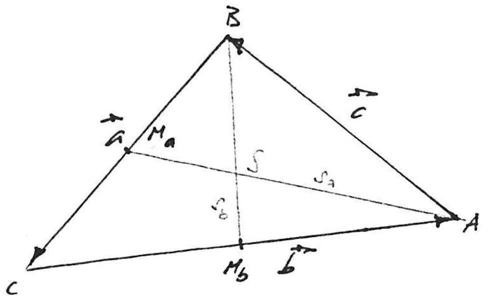
$$\cos\varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{8 + 0 - 15}{\sqrt{41} \cdot \sqrt{13}} = \frac{-7}{23{,}09} = -0{,}303 \Rightarrow \varphi = 107{,}65^\circ$$

$$\varphi^* = 72{,}35^\circ$$

0,5

A6

Bestimme mit Hilfe der Vektorrechnung im Dreieck ΔABC mit $A\,(4|-3)$, $B\,(2|5)$ und $C\,(-3|-4)$ die Geraden der Seitenhalbierenden s_a und s_b in der Parameterform. Berechne anschliessend die Koordinate des Schwerpunktes S.



$$s_a : \vec{r} = \vec{r}_A + \mu\,(\vec{r}_{M_a} - \vec{r}_A)$$

$$s_b : \vec{r} = \vec{r}_B + \lambda\,(\vec{r}_{M_b} - \vec{r}_B)$$

$$\vec{r}_{M_a} = \vec{r}_B + \frac{\vec{a}}{2} \quad / \quad \vec{a} = \vec{r}_C - \vec{r}_B = \begin{pmatrix}-3\\-4\end{pmatrix} - \begin{pmatrix}2\\5\end{pmatrix} = \begin{pmatrix}-5\\-9\end{pmatrix}$$

$$\vec{r}_{M_a} = \begin{pmatrix}2\\5\end{pmatrix} + \begin{pmatrix}-2{,}5\\-4{,}5\end{pmatrix} = \begin{pmatrix}-0{,}5\\0{,}5\end{pmatrix}$$

$$\vec{r}_{M_b} = \vec{r}_C + \frac{\vec{b}}{2} \quad / \quad \vec{b} = \vec{r}_A - \vec{r}_C = \begin{pmatrix}4\\-3\end{pmatrix} - \begin{pmatrix}-3\\-4\end{pmatrix} = \begin{pmatrix}7\\1\end{pmatrix}$$

$$\vec{r}_{M_b} = \begin{pmatrix}-3\\-4\end{pmatrix} + \begin{pmatrix}3{,}5\\0{,}5\end{pmatrix} = \begin{pmatrix}0{,}5\\-3{,}5\end{pmatrix}$$

$$s_a : \vec{r} = \begin{pmatrix}4\\-3\end{pmatrix} + \mu\left[\begin{pmatrix}-0{,}5\\0{,}5\end{pmatrix} - \begin{pmatrix}4\\-3\end{pmatrix}\right] \Rightarrow \underline{\underline{s_a : \vec{r} = \begin{pmatrix}4\\-3\end{pmatrix} + \mu\begin{pmatrix}-9\\7\end{pmatrix}}}$$

$$s_b : \vec{r} = \begin{pmatrix}2\\5\end{pmatrix} + \lambda\left[\begin{pmatrix}0{,}5\\-3{,}5\end{pmatrix} - \begin{pmatrix}2\\5\end{pmatrix}\right] \Rightarrow s_b : \vec{r} = \begin{pmatrix}2\\5\end{pmatrix} + \lambda\begin{pmatrix}-3\\-17\end{pmatrix}$$

$$\underline{\underline{s_b : \vec{r} = \begin{pmatrix}2\\5\end{pmatrix} + \lambda\begin{pmatrix}3\\17\end{pmatrix}}}$$

Berechnung des Koordinatenschwerpunktes ΔABC:

$s_a \cap s_b$

$$\Rightarrow \begin{pmatrix}4\\-3\end{pmatrix} + \mu_s\begin{pmatrix}-9\\7\end{pmatrix} = \begin{pmatrix}2\\5\end{pmatrix} + \lambda_s\begin{pmatrix}3\\17\end{pmatrix}$$

$$\left|\begin{array}{l} 4 - 9\mu_s = 2 + 3\lambda_s \\ -3 + 7\mu_s = 5 + 17\lambda_s \end{array}\right| \Rightarrow \left|\begin{array}{l} -9\mu_s - 3\lambda_s = -2 \\ 7\mu_s - 17\lambda_s = 8 \end{array}\right|$$

$$\Rightarrow \left|\begin{array}{l} 9\mu_s + 3\lambda_s = 2 \\ 7\mu_s - 17\lambda_s = 8 \end{array}\right|$$

Determinantenmethode:

$$\underline{D} = \begin{vmatrix}9 & 3\\7 & -17\end{vmatrix} = -153 - 21 = \underline{-174}$$

$$\underline{D_1} = \begin{vmatrix}2 & 3\\8 & -17\end{vmatrix} = -34 - 24 = \underline{-58}$$

$$\underline{D_2} = \begin{vmatrix}9 & 2\\7 & 8\end{vmatrix} = 72 - 14 = \underline{58}$$

$$\underline{\underline{\mu_s}} = \frac{D_1}{D} = \frac{-58}{-174} = \underline{\underline{\frac{1}{3}}}$$

$$\underline{\underline{\lambda_s}} = \frac{D_2}{D} = \frac{58}{-174} = \underline{\underline{-\frac{1}{3}}}$$

$$S \Rightarrow \vec{r}_S = \begin{pmatrix}4\\-3\end{pmatrix} + \frac{1}{3}\begin{pmatrix}-9\\7\end{pmatrix} = \begin{pmatrix}1\\-2/3\end{pmatrix} \longrightarrow \underline{\underline{S\begin{pmatrix}1\\-2/3\end{pmatrix}}}$$

Kontrolle: $\vec{r}_S = \begin{pmatrix}2\\5\end{pmatrix} - \frac{1}{3}\begin{pmatrix}3\\17\end{pmatrix} = \begin{pmatrix}1\\-2/3\end{pmatrix}$ ok.

A7 Trigonometrische Beziehungen!

a) Formelblatt Additionstheoreme gilt:

$$\cos(3x) = \cos^3(x) - \binom{3}{2}\sin^2(x)\cos(x)$$

$$\cos(3x) = \cos^3(x) - \frac{3!}{(3-2)!\,2!}\sin^2(x)\cos(x)$$

$$\cos(3x) = \cos^3(x) - 3\sin^2(x)\cos(x)$$

mit $\sin^2(x) = 1-\cos^2(x)$ folgt

$$\cos(3x) = \cos^3(x) - 3\left[1-\cos^2(x)\right]\cos(x)$$

$$\cos(3x) = \cos^3(x) - 3\cos(x) + 3\cos^3(x)$$

$$\cos(3x) = 4\cos^3(x) - 3\cos(x) \rightarrow \cos^3(x) = \frac{1}{4}\left[3\cos(x) + \cos(3\right.$$

fehlerfrei

b) $\cos(x) + \sin(x) = \sqrt{2}\sin\left(\frac{\pi}{4}+x\right)$

$$\rightarrow \cos(x) = \sin\left(x+\frac{\pi}{2}\right) \rightarrow \sin\left(x+\frac{\pi}{2}\right) + \sin(x)$$

$$= 2\sin\left(\frac{x+\frac{\pi}{2}+x}{2}\right)\cos\left(\frac{x+\frac{\pi}{2}-x}{2}\right)$$

$$\Rightarrow 2\sin\left(\frac{2x+\frac{\pi}{2}}{2}\right)\cos\left(\frac{\pi}{4}\right)$$

$$= \cancel{2}\sin\left(x+\frac{\pi}{4}\right)\cdot\frac{\sqrt{2}}{\cancel{2}} = \sqrt{2}\sin\left(x+\frac{\pi}{4}\right) \text{ ok.}$$

falscher Ansatz -> richtige Add. -> 0.5

richtiger Ansatz -> 0.5

A8

f_a = Hyperbelfunktion

$$f_a(x) = \frac{a}{x^n} + b$$

Horizontale Asymptote $|x| \to \infty$: $\frac{a}{x^n} \to 0$

für $f_a(x)$ folgt $b = -2$

Pol : $x = 0$ da $\frac{a}{0}$ = nicht definiert!

Wertepaare:

$f_a(-2) = -1 \Rightarrow -1 = \frac{a}{(-2)^n} - 2$

$f_a(2) = -1 \Rightarrow -1 = \frac{a}{2^n} - 2$

$n \in \mathbb{N}$ - Zahlen + gerade!

$f_a(-1) = 2 \Rightarrow 2 = \frac{a}{(-1)^n} - 2$

$f_a(1) = 2 \qquad 2 = \frac{a}{1^n} - 2 \Rightarrow a = 4$

$-1 = \frac{4}{2^n} - 2 \Rightarrow 1 = \frac{4}{2^n} \qquad n = \frac{\log 4}{\log 2} = 2$

$$\Rightarrow f_a(x) = \frac{4}{x^2} - 2$$

1,25

Umkehrfunktion f_a^{-1}

$f_a(x) = \frac{4}{x^2} - 2 \Rightarrow y = \frac{4}{x^2} - 2 \qquad x \leftrightarrow y$

$x = \frac{4}{y^2} - 2$ nach y auflösen!

$y^2 = \frac{4}{x+2}$

$y = \sqrt{\frac{4}{x+2}}$ (Wurzelfunktion)

$$\Rightarrow f_a^{-1} = \sqrt{\frac{4}{x+2}}$$

0,5

A9

$P_1 : f_1 \cap f_2$

$$-4x - 27 = 12x + 21$$
$$16x = -48$$
$$\underline{x = -3} \Rightarrow y = -15 \quad \Big\} \quad \underline{P_1 \begin{pmatrix} -3 \\ -15 \end{pmatrix}}$$

$P_2 : f_1 \cap f_3$

$$-4x - 27 = 4x + 13$$
$$8x = -40$$
$$\underline{x = -5}, \quad y = -7 \quad \Big\} \quad \underline{P_2 \begin{pmatrix} -5 \\ -7 \end{pmatrix}}$$

$P_3 : f_2 \cap f_3$

$$12x + 21 = 4x + 13$$
$$8x = -8$$
$$\underline{x = -1}, \quad y = 9 \quad \Big\} \quad P_3 \begin{pmatrix} -1 \\ 9 \end{pmatrix}$$

Allgemein

$$f(x) = ax^2 + bx + c$$

$$\Rightarrow \left| \begin{array}{l} 9a - 3b + c = -15 \\ 25a - 5b + c = -7 \\ a - b + c = 9 \end{array} \right| \Rightarrow HP: \begin{array}{l} a = 4 \\ b = 28 \\ c = 33 \end{array}$$

$$D = \left| \begin{array}{ccc|cc} 9 & -3 & 1 & 9 & -3 \\ 25 & -5 & 1 & 25 & -5 \\ 1 & -1 & 1 & 1 & -1 \end{array} \right| = -45 - 3 - 25 + 5 + 9 + 75 = \underline{16}$$

$$D_a = \left| \begin{array}{ccc|cc} -15 & -3 & 1 & -15 & -3 \\ -7 & -5 & 1 & -7 & -5 \\ 9 & -1 & 1 & 9 & -1 \end{array} \right| = 75 - 27 + 7 + 45 - 15 - 21 = 64 \Rightarrow \underline{a = 4}$$

$$D_b = \left| \begin{array}{ccc|cc} 9 & -15 & 1 & 9 & -15 \\ 25 & -7 & 1 & 25 & -7 \\ 1 & 9 & 1 & 1 & 9 \end{array} \right| = -63 - 15 + 225 + 7 - 81 + 375 = 448 \Rightarrow \underline{b = 28}$$

$$\Rightarrow \underline{c = 33} \quad (c = -15 - 36 + 84)$$

A9

Parabel: $f(x) = 4x^2 + 28x + 33$

Scheitelform: $f(x) = 4\left(x^2 + 7x + \frac{33}{4}\right)$

$$f(x) = 4\left[(x + 3{,}5)^2 + \frac{33}{4} - 3{,}5^2\right]$$
$$f(x) = 4\left[(x + 3{,}5)^2 - 4\right]$$
$$\underline{\underline{f(x) = 4(x + 3{,}5)^2 - 16}}$$

Kurvendiskussion Parabel

$\cap$-Ordinate $\quad f(0) = 33$

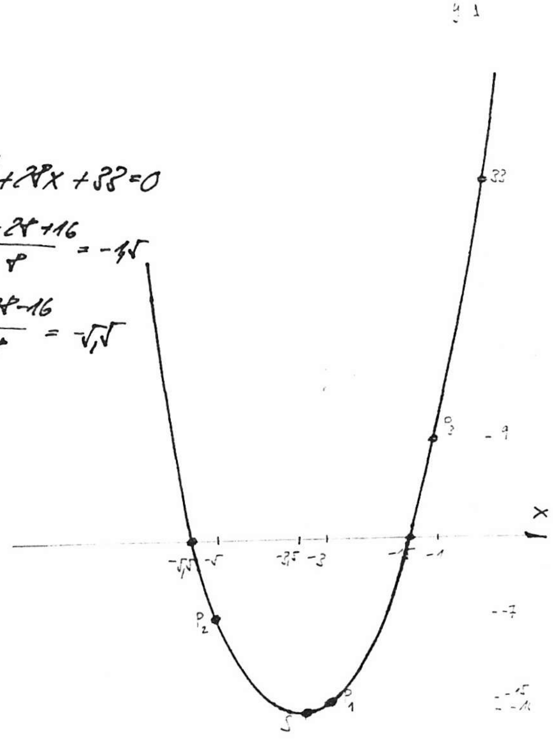
$\cap$-Abszisse $\quad f(x) = 0 \Rightarrow 4x^2 + 28x + 33 = 0$

$$x_{1/2} = \frac{-28 \pm \sqrt{28^2 - 4 \cdot 4 \cdot 33}}{8} \quad x_1 = \frac{-28 + 16}{8} = -1{,}5 \quad x_2 = \frac{-28 - 16}{8} = -5{,}5$$

Scheitelpunkt $\quad S(-u \,|\, v)$

$$\Rightarrow S(-3{,}5 \,|\, -16)$$

$$P_1 \begin{pmatrix} -3 \\ -15 \end{pmatrix}, \quad P_2 \begin{pmatrix} -5 \\ -7 \end{pmatrix}, \quad P_3 \begin{pmatrix} -1 \\ 9 \end{pmatrix}$$



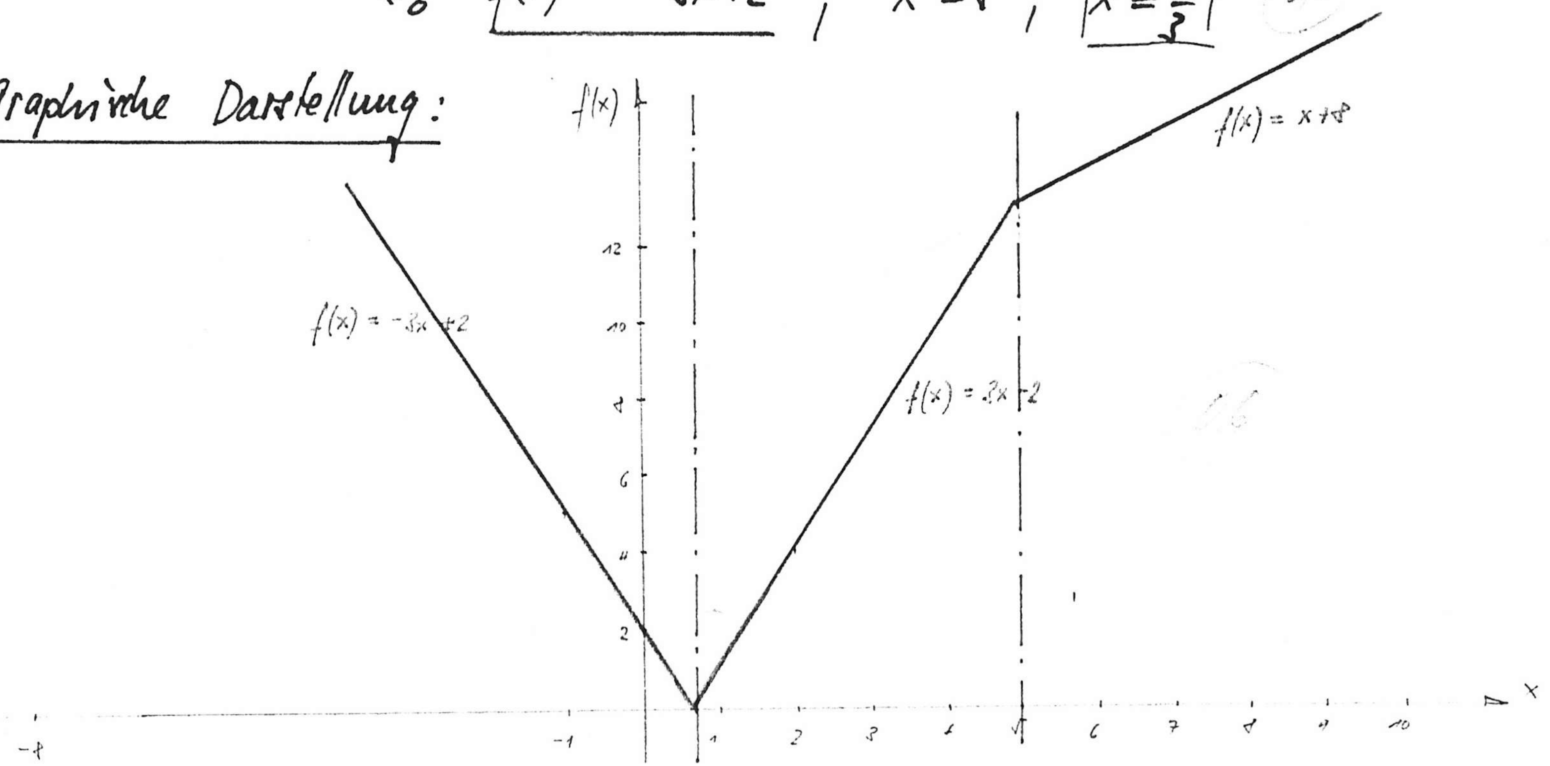
A 10

$$f(x) = |2x - |x-5| + 3| \quad \begin{cases} \xrightarrow{x-5 \geq 0} & f(x) = |2x - x + 5 + 3| = |x+8| \;; \quad x \geq 5 \quad (1) \\ \xrightarrow{x-5 \leq 0} & f(x) = |2x + x - 5 + 3| = |3x - 2| \;; \quad x \leq 5 \quad (2) \end{cases}$$

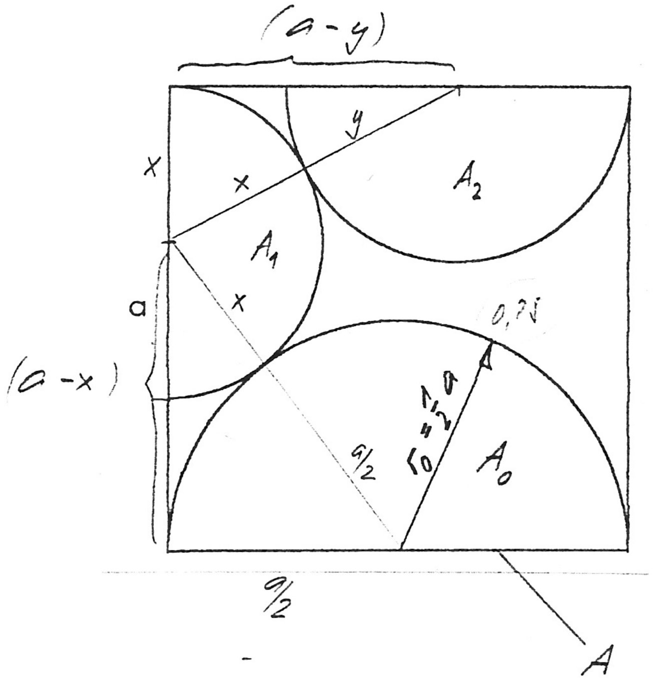
(1) $$f(x) = |x+8| \quad \begin{cases} \xrightarrow{x+8 \geq 0} & \underline{f(x) = x+8} \;; \quad \boxed{x \geq 5} \;; \quad x \geq -8 \\ \xrightarrow{x+8 \leq 0} & f(x) = -x-8 \;; \quad x \geq 5 \;; \quad x \leq -8 \end{cases}$$ → keine Lösung!

(2) $$f(x) = |3x-2| \quad \begin{cases} \xrightarrow{3x-2 \geq 0} & \underline{f(x) = 3x-2} \;; \quad \boxed{x \leq 5 \;; \quad x \geq \frac{2}{3}} \\ \xrightarrow{3x-2 \leq 0} & \underline{f(x) = -3x+2} \;; \quad x \leq 5 \;; \quad \boxed{x \leq \frac{2}{3}} \end{cases}$$ (Intervall)

<u>graphische Darstellung:</u>



A11



Pythagoras:

$$\left(\frac{a}{2}+x\right)^2 = \left(\frac{a}{2}\right)^2 + (a-x)^2$$

$$\cancel{\frac{a^2}{4}} + ax + \cancel{x^2} = \cancel{\frac{a^2}{4}} + a^2 - 2ax + \cancel{x^2} \quad /$$

$$x = a - 2x$$

$$3x = a$$

$$x = \frac{1}{3}a \qquad 1$$

oder $\underline{\underline{r_1 = \frac{1}{3}a}}$ mit $r_1 = x$

$$(x+y)^2 = x^2 + (a-y)^2 \quad \text{mit } x = \frac{1}{3}a \text{ folgt}$$

$$\left(\frac{a}{3}+y\right)^2 = \left(\frac{a}{3}\right)^2 + (a-y)^2$$

$$\cancel{\frac{a^2}{9}} + \frac{2}{3}ay + \cancel{y^2} = \cancel{\frac{a^2}{9}} + a^2 - 2ay + \cancel{y^2} \quad / : a$$

$$\frac{2}{3}y = a - 2y$$

$$\frac{8y}{3} = a \quad \rightarrow \quad y = \frac{3}{8}a \quad \text{mit } y = r_2$$

$$\underline{\underline{r_2 = \frac{3}{8}a}} \qquad 1$$

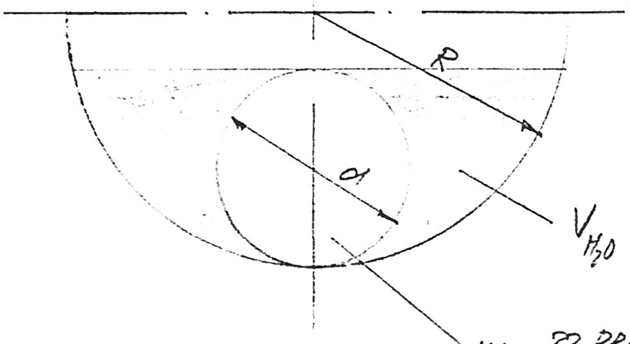
$$A = a^2 = 100\%$$

$$A_\Sigma = A_0 + A_1 + A_2 = \frac{\left(\frac{a}{2}\right)^2\pi}{2} + \frac{\left(\frac{a}{3}\right)^2\pi}{2} + \frac{\left(\frac{3}{8}a\right)^2\pi}{2}$$

$$= \frac{\pi}{2}\left(\frac{a^2}{4} + \frac{a^2}{9} + \frac{9a^2}{64}\right) = 0{,}788\,a^2$$

$$\underline{\underline{A_\Sigma}} = \frac{0{,}788\,a^2 \cdot 100\%}{a^2} = \underline{\underline{78{,}8\,\%}} \qquad 0{,}75$$

A12



$V_{H_2O} = 200\,dm^3$

$m = 32{,}88\,kg$, $\rho_{St} = 7{,}85\,kg/dm^3$

$$\rightarrow \underline{V} = \frac{m}{\rho_{St}} = \frac{32{,}88\,kg}{7{,}85\,kg/dm^3} = \underline{4{,}19\,dm^3}$$

gesamtes Volumen im Becken $V_{tot} = 200\,dm^3 + 4{,}19\,dm^3$

$$\underline{\underline{V_{tot} = 204{,}19\,dm^3}}$$

V_{tot} entspricht einem <u>Kugelabschnitt</u>.

$$V = \frac{\pi h^2}{3}(3R - h) \qquad \text{mit } h = d \rightarrow \text{Bild}$$

$$V = \frac{\pi d^2}{3}(3R - d)$$

$$R = \frac{\frac{3V}{d^2\pi} + d}{3} = \frac{V}{d^2\pi} + \frac{d}{3}$$

$$\underline{\underline{R}} = \frac{204{,}19\,dm^3}{(2\,dm)^2\pi} + \frac{2\,dm}{3} = \underline{\underline{16{,}92\,dm}}$$

<u>Bem</u>: „Gefüllt" bedeutet nicht „Randvoll"