1 (3P)

$$T(y) = \left(-3y^2 + \frac{1}{3y^3}\right)^4 \quad ; \; y \in \mathbb{R}^*$$

$$T(y) = \binom{4}{0}\left(-3y^2\right)^4 + \binom{4}{1}\left(-3y^2\right)^3\left(\frac{1}{3y^3}\right) + \binom{4}{2}\left(-3y^2\right)^2\left(\frac{1}{3y^3}\right)^2 + \binom{4}{3}\left(-3y^2\right)\left(\frac{1}{3y^3}\right)^3$$

$$+ \binom{4}{4}\left(\frac{1}{3y^3}\right)^4$$

• aufstellen 5 × 0,2P

$$= \frac{\cancel{4!}}{\cancel{0!\,4!}}_{1} \cdot 81 \cdot y^8 + \frac{\cancel{4!}}{\cancel{1!\,3!}}_{4}\left(-\cancel{27}^{-9}\right)y^{\cancel{6}\,3} \cdot \frac{1}{\cancel{3}_{1}\,y^{\cancel{3}}} + \frac{\cancel{4!}}{\cancel{2!\,2!}}_{6} \cdot \cancel{9}^{1}\cancel{y^4} \cdot \frac{1}{\cancel{9}_{1}\,y^{\cancel{6}\,2}} + \frac{\cancel{4!}}{\cancel{3!\,1!}}_{4} \cdot \left(-\cancel{3}^{1}\right)y^{\cancel{2}} \cdot \frac{1}{\cancel{27}_{-9}\,y^{\cancel{9}\,7}}$$

$$+ \frac{\cancel{4!}}{\cancel{4!\,0!}}_{1} \cdot \frac{1}{81y^{12}}$$

• ausrechnen
Kürzen und Vereinfachen
5 × 0,4P

$$T(y) = 81y^8 - 36y^3 + 6 \cdot \frac{1}{y^2} - \frac{4}{9} \cdot \frac{1}{y^7} + \frac{1}{81y^{12}}$$

$$T(y) = 81y^8 - 36y^3 + \frac{6}{y^2} - \frac{4}{9y^7} + \frac{1}{81y^{12}}$$

• Abzüge prop.

2(3P) $\quad f(x) = b^{(x+c)} + d \quad \wedge \quad b \in \mathbb{R}^+ \setminus \{1\}$

$c, d \in \mathbb{R}$

$x \to -\infty \quad \wedge \quad f(x) \to -1$

$\Rightarrow b^{-\infty} = 0 \Rightarrow \underline{\underline{d = -1}}$ (0,4P)

$f(x) = b^{(x+c)} - 1$

$P_1: \quad 0 = b^{(1+c)} - 1 \Rightarrow 1 = b^{(1+c)}$

$P_2: \quad 2 = b^{(2+c)} - 1 \qquad \Rightarrow b^0 = 1 \Rightarrow 1 + c = 0$

(0,6P) $\underline{c = -1}$

$\Rightarrow \quad 2 = b^{(2-1)} - 1$

(0,6P)

$b^1 = 3 \Rightarrow \underline{\underline{b = 3}} \quad \Rightarrow \underline{\underline{f(x) = 3^{(x-1)} - 1}}$

<u>Kontrolle</u>

<u>Ab-Ordinate</u> $\Rightarrow \underline{\underline{f(0)}} = 3^{-1} - 1 = \underline{\underline{-\frac{2}{3}}}$

<u>Umkehrfunktion</u>

$\underline{\underline{D_{f^{-1}} = W_f = \mathbb{R}^{>-1}}}$ (0,4P)

$y = 3^{(x-1)} - 1$

$\underline{\underline{W_{f^{-1}} = D_f = \mathbb{R}}}$ (0,4P)

$y + 1 = 3^{(x-1)} \quad | \log_3()$

$\log_3(y+1) = x - 1$ $\qquad$ · Abzüge prop.

$x = \log_3(y+1) + 1$

(0,6P)

$\underline{\underline{y = f^{-1}(x) = \log_3(x+1) + 1}}$

3 (3P)

$$\frac{1}{\sqrt{4x+1}} + \frac{1}{\sqrt{3x-2}} = \frac{\sqrt{13x-1}}{\sqrt{12x^2-5x-2}}$$

(1) (2) (3) (4)

$\mathbb{D}$:

(1) $4x+1 > 0 \rightarrow x > -\frac{1}{4}$

(2) $3x-2 > 0 \rightarrow x > \frac{2}{3}$

(4) $13x-1 \geq 0 \rightarrow x \geq \frac{1}{13}$

(3) $12x^2-5x-2 = 0$

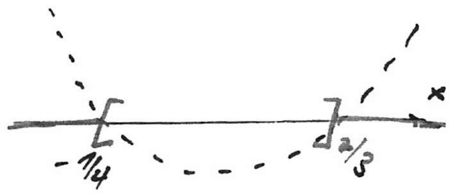
$(4x+1)(3x-2) = 0 \rightarrow x_1 = -\frac{1}{4}$, $x_2 = \frac{2}{3}$

$\rightarrow (4x+1)(3x-2) > 0$

$\rightarrow \mathbb{D} = \mathbb{R}^{> 2/3}$

• 4 × 0,2P

• Verknüpfung 0,2P



1P

$\rightarrow \left(\sqrt{3x-2} + \sqrt{4x+1}\right)^2 = \left(\sqrt{13x-1}\right)^2$

$3x-2 + 2\sqrt{3x-2}\sqrt{4x+1} + 4x+1 = 13x-1$

$2\sqrt{3x-2}\sqrt{4x+1} = 6x$

$\left(\sqrt{3x-2}\sqrt{4x+1}\right)^2 = (3x)^2$

$12x^2-5x-2 = 9x^2$

$3x^2-5x-2 = 0$

$\rightarrow x_{1/2} = \frac{5 \pm \sqrt{25+4\cdot3\cdot2}}{6}$

$x_{1/2} = \frac{5 \pm 7}{6}$ — $x_1 = 2 \rightarrow$ nach $\mathbb{D}$ ok!

— $x_2 = -\frac{1}{3}$

1,5P

• Fehler 0,3P

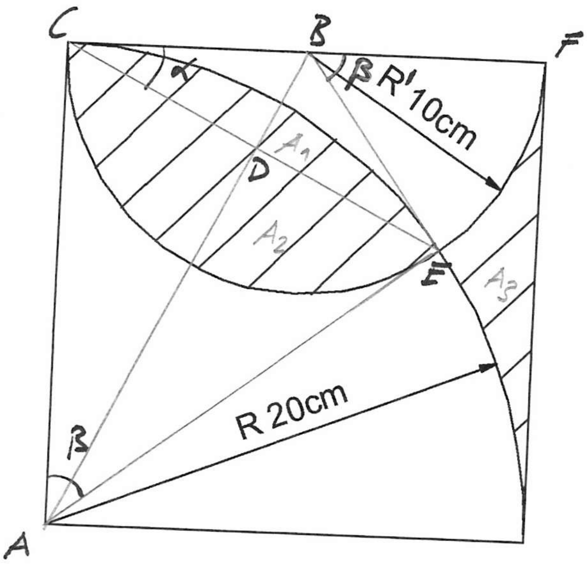
Kontrolle

$$\frac{1}{\sqrt{9}} + \frac{1}{\sqrt{4}} \stackrel{?}{=} \frac{\sqrt{25}}{\sqrt{36}}$$

Test 0,5P

$$\frac{1}{3} + \frac{1}{2} \stackrel{?}{=} \frac{5}{6} \quad \text{ok} \quad \Rightarrow \mathbb{L} = \{2\}$$

4 (3P)



$\alpha = \frac{\beta}{2}$

$\triangle ABC \sim \triangle CDB$

$\overline{AB} = \sqrt{20^2 + 10^2}$

$\underline{\overline{AB} = 22{,}36\ cm}$

$\frac{\overline{BC}}{\overline{CD}} = \frac{\overline{AB}}{\overline{AC}} \Rightarrow \overline{CD} = \overline{BC} \cdot \frac{\overline{AC}}{\overline{AB}}$

$\overline{CD} = 10 \cdot \frac{20}{22{,}36}$

$\underline{\overline{CD} = 8{,}94\ cm}$

$\sin\alpha = \sin\frac{\beta}{2} = \frac{\overline{CD}}{AC} = \frac{8{,}94}{20} = 0{,}447$

$\underline{\alpha = 26{,}57°}$ $\quad \underline{\beta = 53{,}13°}$

$\Rightarrow \overline{CE} = 2\overline{CD}$

$\underline{\overline{CE} = 17{,}89\ cm}$

$A_1 = \frac{R^2 \cdot \pi \cdot \beta}{360} - \frac{1}{2} R^2 \cdot \sin\beta$

• 10 Resultate à 0,3P
• Abzüge prop
+ Strategie

$\underline{A_1} = \frac{20^2 \pi \cdot 53{,}13}{360} - \frac{1}{2} \cdot 20^2 \sin 53{,}13 = \underline{25{,}46\ cm^2}$

$\underline{\overline{BD}} = \overline{BC} \cdot \sin\alpha = 10 \cdot \sin 26{,}57 = \underline{4{,}47\ cm}$

$A_2 = \frac{R'^2 \pi}{2} - \frac{R'^2 \pi \cdot \beta}{360} - A_{CEB}$

$A_2 = \frac{10^2 \pi (180 - 53{,}13)}{360} - \frac{17{,}89 \cdot 4{,}47}{2} = 110{,}71 - 40{,}0$

$\underline{A_2 = 70{,}71\ cm^2}$

$A_3 = a^2 - \frac{R^2 \pi}{4} - \frac{R'^2 \pi \cdot \beta}{360} - A_{CEB} + A_1$

$A_3 = 20^2 - \frac{20^2 \pi}{4} - \frac{10^2 \pi \cdot 53{,}13}{360} - 40{,}0 + 25{,}46$

$\underline{A_3 = 24{,}94\ cm^2}$

$\underline{\underline{A}} = \sum_{i=1}^{3} A_i = \underline{\underline{121{,}11\ cm^2}}$

5 (3P) $E_1: \quad x - 2y + z + 3 = 0$

$E_2: \quad x + y - 3z - 2 = 0$

Die Gerade (Schnittgerade) kann nicht zu allen Koordinatenachsen parallel sein. Ich wähle die z-Koordinate als Parameter λ. Falls die Schnittgerade parallel zur xy-Ebene ist, wählen wir eine andere Koordinate als Parameter.

$\Rightarrow E_1: \quad x - 2y + \lambda + 3 = 0$

$$2y = x + \lambda + 3$$

$$y = \frac{x + \lambda + 3}{2} \longrightarrow E_2$$

0,8P

$$E_2: \quad x + \frac{x + \lambda + 3}{2} - 3\lambda - 2 = 0$$

$$2x + x + \lambda + 3 - 6\lambda - 4 = 0$$

$$3x = 5\lambda + 1$$

$$\underline{x = \frac{1}{3} + \frac{5}{3}\lambda}$$

0,8P

$$\Rightarrow y = \frac{\frac{1 + 5\lambda}{3} + \lambda + 3}{2}$$

$$y = \frac{1 + 5\lambda + 3\lambda + 9}{6} = \frac{10}{6} + \frac{8}{6}\lambda$$

$$\underline{y = 5/3 + 4/3\,\lambda}$$

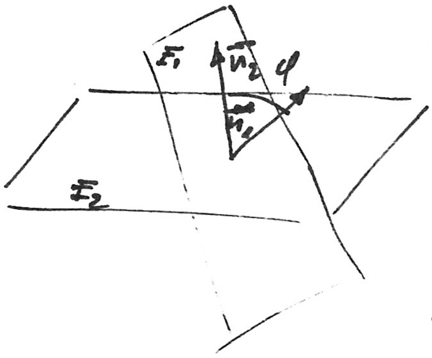
0,8P

$$\underline{z = \lambda}$$

$$\Rightarrow g: \quad \vec{r} = \begin{pmatrix} 1/3 \\ 5/3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 5/3 \\ 4/3 \\ 1 \end{pmatrix}$$

. Fehler 0,5P

$$\underline{\underline{g: \quad \vec{r} = \begin{pmatrix} 1/3 \\ 5/3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}}}$$



$$\cos\varphi = \frac{\vec{n}_1 \cdot \vec{n}_2}{n_1 \cdot n_2} = \frac{\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}}{\sqrt{6} \cdot \sqrt{11}} = \frac{-4}{8,124}$$

$$E_1: \vec{n}_1 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$E_2: \vec{n}_2 = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$$

0,6P

$$\cos\varphi = -0,492$$

$$\underline{\varphi = 119,50^\circ}$$

$$\underline{\underline{\varphi' = 60,5^\circ}}$$

6 (3P) Version 1

$$4\sin(x)\cos(x) = -1{,}2 \quad \wedge \quad D_{max} \subseteq [0;2\pi]$$

$$2\sin(x)\cos(x) = \sin(2x) \quad 0{,}5P$$

$$\Rightarrow \quad 2\sin(2x) = -1{,}2$$

$$\sin(2x) = -0{,}6$$

$$2x = -0{,}6435$$

$$\underline{x = -0{,}3218} \quad 0{,}5P$$

Lösungen im Intervall $[0;2\pi]$

Periodenlänge: $\underline{P} = \frac{2\pi}{|\omega|} = \frac{2\pi}{2} = \underline{\pi}$ 0,4P

$$x_{1/2} = x + k\cdot\pi \quad \wedge \quad k \in \mathbb{Z}$$

$$\Rightarrow \quad x_1 = -0{,}3218 + \pi \longrightarrow \underline{\underline{x_1 = 2{,}8198}} \quad 0{,}4P$$

$$x_2 = -0{,}3218 + 2\pi \longrightarrow \underline{\underline{x_2 = 5{,}9614}} \quad 0{,}4P$$

$$x_{3/4} = \left(\frac{\pi}{2} - x\right) + k\pi \quad \wedge \quad k \in \mathbb{Z}$$

$$x_3 = \frac{\pi}{2} - (-0{,}3218) \longrightarrow \underline{\underline{x_3 = 1{,}893}} \quad 0{,}4P$$

$$x_4 = \frac{\pi}{2} - (-0{,}3218) + \pi \longrightarrow \underline{\underline{x_4 = 5{,}0341}}$$

0,4P

$$\underline{\underline{\mathbb{L} = \{1{,}89;\ 2{,}82;\ 5{,}03;\ 5{,}96\}}}$$

• Abzüge prop.

6 (3P) Version 2

$$4\sin(x)\cos(x) = -1{,}2 \qquad \wedge \quad D_{max} \triangleq [0;2\pi]$$

$$\left(4\sin(x)\sqrt{1-\sin^2(x)}\right)^2 = (-1{,}2)^2$$

$$16\sin^2(x)\,(1-\sin^2(x)) = 1{,}44$$

$$16\sin^2(x) - 16\sin^4(x) = 1{,}44$$

$$\Rightarrow \quad 16\sin^4(x) - 16\sin^2(x) + 1{,}44 = 0 \qquad 0{,}8P$$

Substitution $\quad \sin^2(x) = a \qquad 0{,}2P$

$$\Rightarrow \quad 16a^2 - 16a + 1{,}44 = 0$$

$$a_{1/2} = \frac{16 \pm \sqrt{256 - 4\cdot 16\cdot 1{,}44}}{32} \quad \begin{cases} a_1 = 0{,}9 \\ a_2 = 0{,}1 \end{cases} \qquad 0{,}4P$$

$$|\sin(x)| = \pm\sqrt{0{,}9} = \pm\,0{,}9487$$

$$x_{1/2} = \pm\,1{,}249 \qquad 0{,}2P$$

$$|\sin(x)| = \pm\sqrt{0{,}1} = \pm\,0{,}3162$$

$$x_{3/4} = \pm\,0{,}322 \qquad 0{,}2P$$

Mögliche Lösungen (Periodizität berücksichtigen)

$0{,}322 \;/\; 1{,}249 \;/\; \pi - 1{,}249 \;/\; \pi - 0{,}322 \;/\; \pi - (-0{,}322) \;/\; \pi - (-1{,}249) \;/$

$2\pi - 0{,}322 \;/\; 2\pi - 1{,}249$

$\Rightarrow \quad 0{,}322 \;/\; 1{,}249 \;/\; \underline{1{,}893} \;/\; \underline{2{,}820} \;/\; 3{,}463 \;/\; 4{,}391 \;/\; \underline{5{,}034} \;/\; \underline{5{,}961}$ $\qquad 1{,}2P$

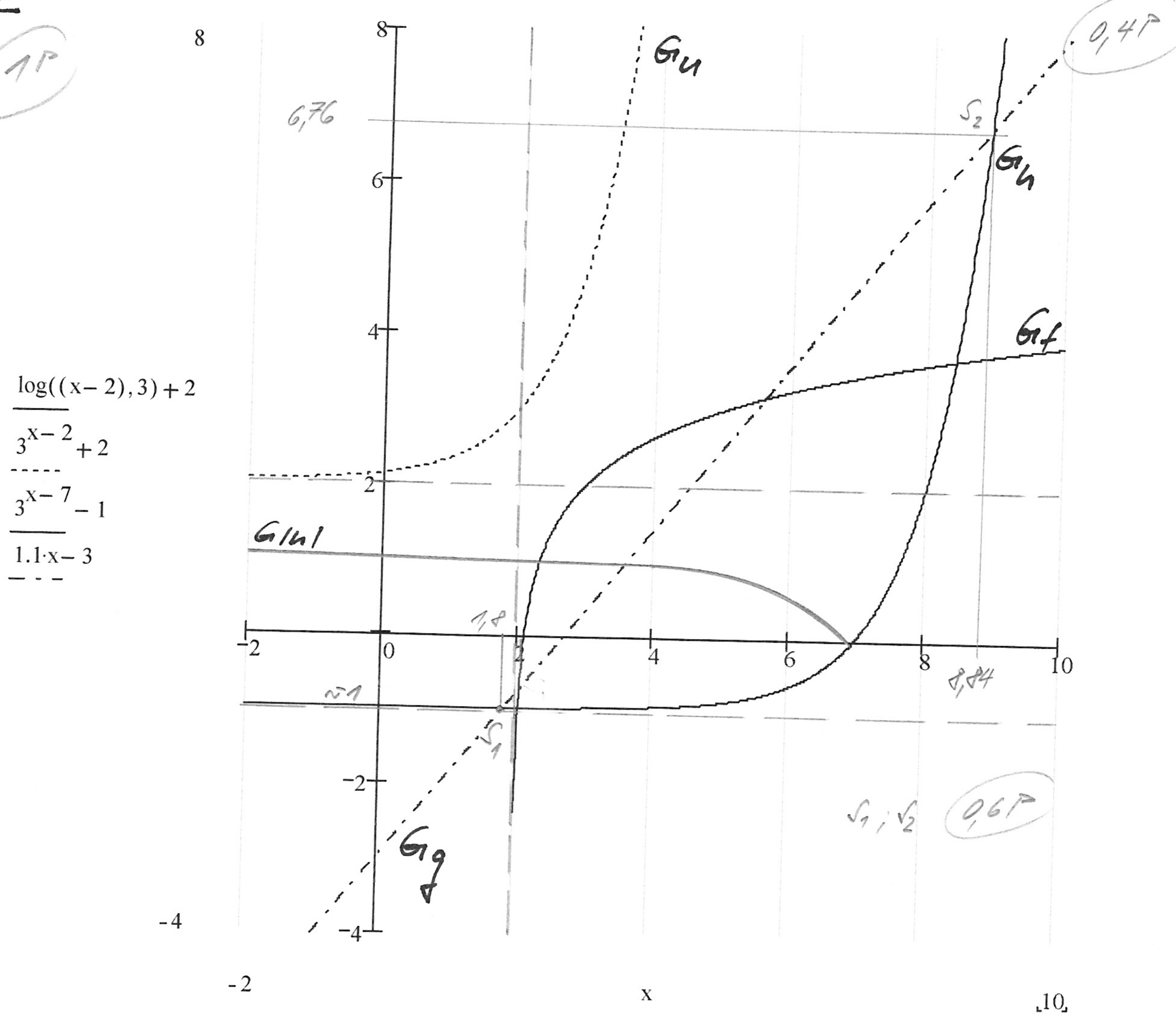
Kontrolle wegen des Lösungsgangs!

$$\Rightarrow \quad \mathbb{L} = \{1{,}893;\ 2{,}82;\ 5{,}034;\ 5{,}961\}$$

7(5P)

VII

1P



I **Nullstelle f(x)**

$\rightarrow \quad f(x) = 0 \quad \rightarrow \quad \log_3(x-2) + 2 = 0$

$\log_3(x-2) = -2 \quad / 3^{()}$

$x - 2 = 3^{-2}$

$\underline{\underline{x = 2,\bar{1}}}$ 0,5P

II **Wertebereich**

$\underline{\underline{W_f = \mathbb{R}}}$ 0,3P

• abfüge prop.

Fortsetzung 7

III Umkehrfunktion $u(x)$

$$f(x) = \log_3(x-2) + 2$$

$$\rightarrow \quad y = \log_3(x-2) + 2$$

$$y - 2 = \log_3(x-2) \qquad / \, 3^{(\,)}$$

$$3^{y-2} = x - 2$$

1P

$$x = 3^{y-2} + 2 \quad \rightarrow \quad \underline{\underline{u(x) = 3^{x-2} + 2}}$$

IV $u(x)$

$$\underline{\underline{\mathbb{D}_u = \mathbb{W}_f = \mathbb{R}}}$$

0,3P

$$\underline{\underline{\mathbb{W}_u = \mathbb{D}_f = \mathbb{R}^{>2}}}$$

0,3P

V $\vec{v} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} \quad \rightarrow \quad h(x) = 3^{x-2-5} + 2 - 3$

$$\underline{\underline{h(x) = 3^{x-7} - 1}}$$

0,6P

VI $g(x) = 1{,}1x - 3 \quad , \quad \mathbb{D}_g = \mathbb{R}$

$$\rightarrow \quad 1{,}1x - 3 = 3^{x-7} - 1$$

$$3^{x-7} - 1{,}1x + 2 = 0 \quad \rightarrow \quad x_1 = 1{,}8213$$

$$\rightarrow \quad x_2 = 8{,}8639$$

$$\underline{\underline{S_1 \begin{pmatrix} 1{,}8213 \\ -0{,}9966 \end{pmatrix} \quad S_2 \begin{pmatrix} 8{,}8639 \\ 6{,}7503 \end{pmatrix}}}$$

· Punktzahl im Diagramm

VII siehe Diagramm

8 (6P)

I $g \subset E$

$$g: \vec{r} = \underbrace{\begin{pmatrix}1\\1\\1\end{pmatrix}}_{\vec{r}_P} + \lambda \underbrace{\begin{pmatrix}3\\-2\\-1\end{pmatrix}}_{\vec{a}} \qquad E: x+y+z-3=0$$

$$\Rightarrow \vec{n} = \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$g \subset E$: $\quad \vec{a}\cdot\vec{n} = 0 \quad \wedge \quad \vec{n}\cdot\vec{r}_P - 3 \overset{!}{=} 0$

$$\underline{\underline{\begin{pmatrix}3\\-2\\-1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = 3-2-1=0}} \qquad 0{,}5P$$

$\wedge$

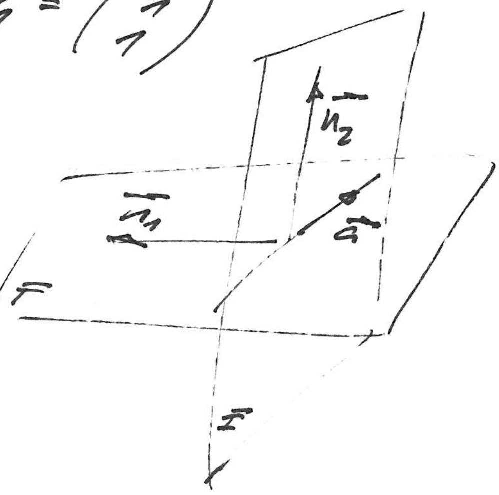
$$\underline{\underline{\begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} - 3 = 3-3=0}} \qquad 0{,}5P$$

• Abzüge prop.

II $F \perp E \quad \wedge \quad F \cap E = g$

$$E: x+y+z-3=0 \quad \rightarrow \quad \vec{n}_1 = \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$g: \vec{r} = \underbrace{\begin{pmatrix}1\\1\\1\end{pmatrix}}_{\vec{r}_P} + \lambda \underbrace{\begin{pmatrix}3\\-2\\-1\end{pmatrix}}_{\vec{a}}$$



$$\underline{\vec{n}_2} = \vec{a}\times\vec{n}_1 = \begin{pmatrix}3\\-2\\-1\end{pmatrix} \times \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$= \begin{pmatrix} \begin{vmatrix}-2 & 1\\-1 & 1\end{vmatrix} \\ -\begin{vmatrix}3 & 1\\-1 & 1\end{vmatrix} \\ \begin{vmatrix}3 & 1\\-2 & 1\end{vmatrix} \end{pmatrix} = \underline{\begin{pmatrix}-1\\-4\\5\end{pmatrix}} \qquad 1P$$

• Abzüge prop.

$$F: \quad \vec{n}_2(\vec{r} - \vec{r}_P) = 0$$

$$\begin{pmatrix}-1\\-4\\5\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} - \underbrace{\begin{pmatrix}-1\\-4\\5\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}}_{=0} = 0 \Rightarrow -x-4y+5z=0$$

$$F: \underline{\underline{x+4y-5z=0}}$$

Fortsetzung 8

III $E: x+y+z-3=0$

Eine Ebene kann nicht zu allen drei Koordinatenachsen parallel sein. Ich wähle deshalb die Koordinaten x und y eines auf der Ebene liegenden Punktes als Parameter λ und μ.

$$\left.\begin{array}{l} x=\lambda \\ y=\mu \end{array}\right\} \Rightarrow \lambda+\mu+z-3=0$$

$$z=3-\lambda-\mu$$

$$E: \vec{r}=\begin{pmatrix}0\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}1\\0\\-1\end{pmatrix}+\mu\begin{pmatrix}0\\1\\-1\end{pmatrix}$$

1P

IV $G \perp xy\text{-Ebene} \wedge g \subset G$

$xy\text{-Ebene}: \quad z=0 \Rightarrow \vec{n}_1=\begin{pmatrix}0\\0\\1\end{pmatrix}$

$G: \vec{n}_2(\vec{r}-\vec{r}_P)=0$

$$g: \vec{r}=\underbrace{\begin{pmatrix}1\\1\\1\end{pmatrix}}_{\vec{r}_P}+\lambda\underbrace{\begin{pmatrix}3\\-2\\-1\end{pmatrix}}_{\vec{a}}$$

1P

$$\underline{\vec{n}_2}=\vec{n}_1\times\vec{a}=\begin{pmatrix}0\\0\\1\end{pmatrix}\times\begin{pmatrix}3\\-2\\-1\end{pmatrix}=\begin{pmatrix}\begin{vmatrix}0&-2\\1&-1\end{vmatrix}\\-\begin{vmatrix}0&3\\1&-1\end{vmatrix}\\\begin{vmatrix}0&3\\0&-2\end{vmatrix}\end{pmatrix}=\underline{\begin{pmatrix}2\\3\\0\end{pmatrix}}$$

$$G: \begin{pmatrix}2\\3\\0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}-\begin{pmatrix}2\\3\\0\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}=0$$

· Abkürze prop.

$$2x+3y-(2+3+0)=0$$

$$\underline{G: 2x+3y-5=0}$$

Fortsetzung 8

V

$$g: \vec{r} = \begin{pmatrix}1\\1\\1\end{pmatrix} + \lambda \begin{pmatrix}3\\-2\\-1\end{pmatrix}$$

$$h: \vec{r} = \begin{pmatrix}7\\6\\5\end{pmatrix} + \mu \begin{pmatrix}1\\0\\-1\end{pmatrix}$$

Parallelität:
$$\left.\begin{array}{l} \vec{a} = \begin{pmatrix}3\\-2\\-1\end{pmatrix} \\ \vec{b} = \begin{pmatrix}1\\0\\-1\end{pmatrix} \end{array}\right\} \vec{a} \nparallel \vec{b} \rightarrow \text{keine Identität}$$

0,5P

$\Rightarrow$ windschief

$$D(\vec{a}, \vec{b}, \vec{r}_1 - \vec{r}_2) = \begin{vmatrix} 3 & 1 & 6 \\ -2 & 0 & 5 \\ -1 & -1 & 4 \end{vmatrix} = \underline{30}$$

0,5P

$$\vec{r}_1 - \vec{r}_2 = \begin{pmatrix}1\\1\\1\end{pmatrix} - \begin{pmatrix}7\\6\\5\end{pmatrix} = \begin{pmatrix}-6\\-5\\-4\end{pmatrix} \Rightarrow \vec{r}_2 - \vec{r}_1 = \begin{pmatrix}6\\5\\4\end{pmatrix}$$

$\Rightarrow D \neq 0 \Rightarrow$ $\underline{\underline{g \wedge h \text{ sind windschief}}}$

VI

$$h: \vec{r} = \underbrace{\begin{pmatrix}7\\6\\5\end{pmatrix}}_{\vec{r}_P} + \mu \underbrace{\begin{pmatrix}1\\0\\-1\end{pmatrix}}_{\vec{a}}$$

$E: x + y + z - 3 = 0$

$$\rightarrow \vec{n} = \begin{pmatrix}1\\1\\1\end{pmatrix}$$

$\vec{n} \circ \vec{a} = 0 \Rightarrow \begin{pmatrix}1\\1\\1\end{pmatrix} \circ \begin{pmatrix}1\\0\\-1\end{pmatrix} = 1 + 0 - 1 = 0$ $\underline{\text{ok}}$

$\vec{r}_P \notin E$

$$\Rightarrow \vec{n} \cdot \vec{r}_P - 3 = \begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}7\\6\\5\end{pmatrix} - 3 = \underline{\underline{7 + 6 + 5 - 3 \neq 0}}$$

$\rightarrow$ $\underline{h \parallel E!}$

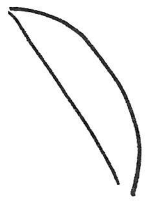
$$d = \frac{\vec{n} \circ \vec{r} - 3}{|\vec{n}|} = \frac{\begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}7\\6\\5\end{pmatrix} - 3}{\sqrt{3}}$$

1P

$$\underline{\underline{d}} = \frac{15}{\sqrt{3}} = \underline{\underline{8{,}66}}$$

• Abprüfen prop.

9 (3P)

 $\rightarrow \quad A_1 = \frac{r^2\pi}{6} - \frac{r^2\sqrt{3}}{4}$ 0,6P

Halbkreis: $\rightarrow \quad A_2 = \frac{r^2\pi}{8} \quad \wedge \quad A_0 = \frac{\cancel{4}r^2\pi}{\cancel{4}}$

$$A = A_0 - 3(A_1 + A_2)$$ → 0,4P

$$A = r^2\pi - 3\left[\left(\frac{r^2\pi}{6} - \frac{r^2\sqrt{3}}{4} + \frac{r^2\pi}{8}\right)\right]$$

$$A = r^2\pi - \frac{\cancel{3}^1 r^2\pi}{\cancel{6}2} + \frac{3\sqrt{3}r^2}{4} - \frac{3r^2\pi}{8}$$

$$A = \frac{8r^2\pi - 4r^2\pi - 3r^2\pi}{8} + \frac{3\sqrt{3}r^2}{4}$$

$$\boxed{A = \frac{r^2\pi}{8} + \frac{3\sqrt{3}r^2}{4} = r^2\left(\frac{\pi}{8} + \frac{3\sqrt{3}}{4}\right)}$$ 1,1P

$$A = 0,5^2\left[\left(\frac{\pi}{8}\right) + \frac{3\sqrt{3}}{4}\right] = \underline{0,423\,dm^2}$$ 0,3P

$$V = A \cdot l = 0,423 \cdot 20 = \underline{8,459\,dm^3}$$ 0,3P

$$m = V \cdot \rho = 8,459 \cdot 2,7 = \underline{\underline{22,84\,kg}}$$ 0,3P

10 (3P) $\quad f: x \rightarrow f(x)$, $D_f = \mathbb{R}$, $f(x) = |2x - |x-5| + 3|$

Intervall : $[-5; 10]$

I $\quad$ pos. Fall : $x - 5 \geq 0 \Rightarrow x \geq 5$

$$f(x) = |2x - |x-5| + 3| = |2x - x + 5 + 3| = |x+8|$$

$f(x) = |x+8|$ $\quad$ $x+8 \geq 0 \Rightarrow x \geq -8 \Rightarrow \underline{f_1(x) = x+8 \wedge [5; \infty[}$ $\quad$ 0,1 $\quad$ 0,2

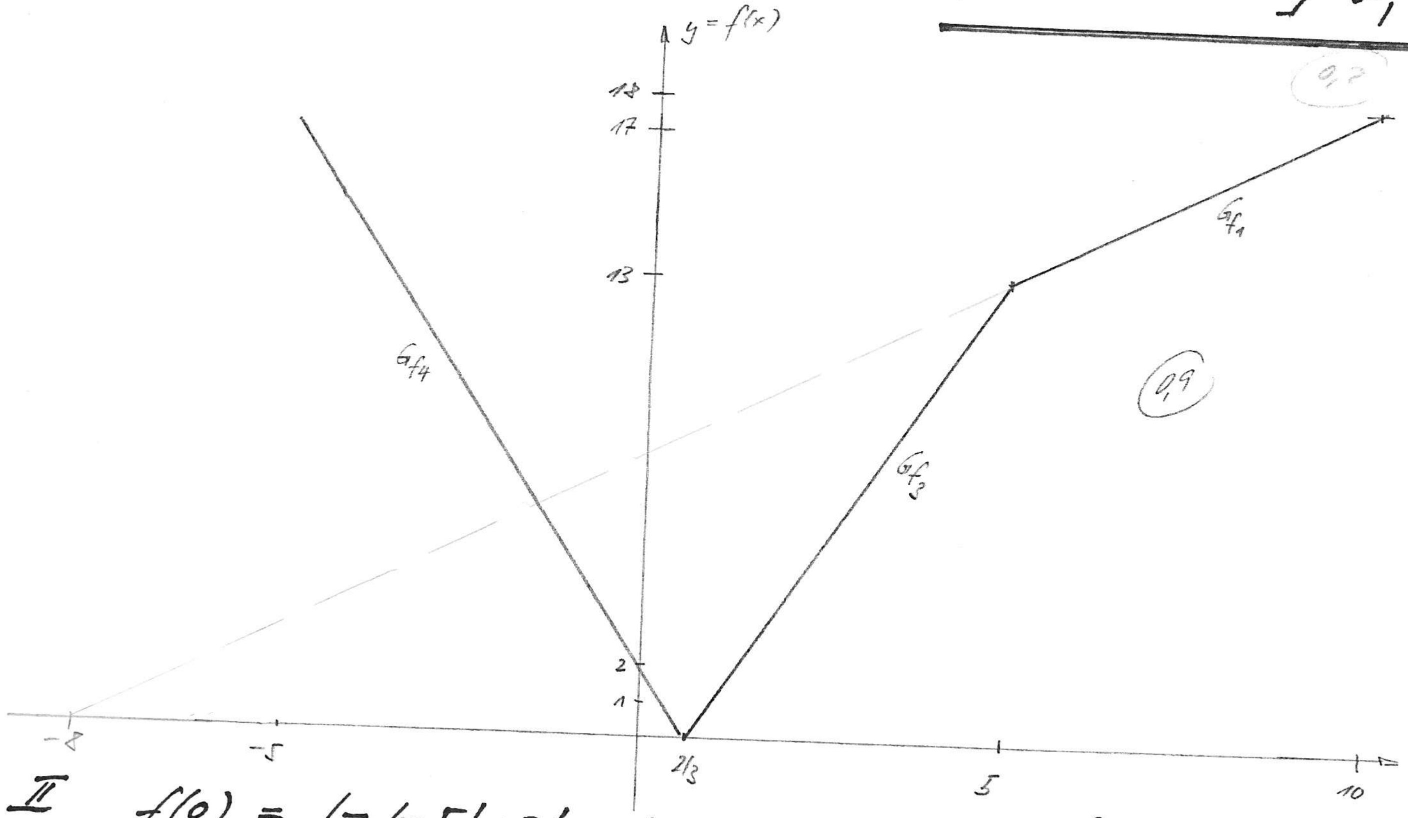
0,2 $\quad$ $x+8 \leq 0 \Rightarrow x \leq -8 \Rightarrow \underline{f_2(x) = -x-8} \wedge \underline{D = \{\}}$ $\quad$ 0,1 $\quad$ 0,2

neg. Fall : $x - 5 \leq 0 \quad x \leq 5$

$$f(x) = |2x + x - 5 + 3| = |3x - 2|$$

$f(x) = |3x-2|$ $\quad$ $3x - 2 \geq 0 \Rightarrow x \geq \frac{2}{3} \Rightarrow \underline{f_3(x) = 3x - 2 \wedge [\frac{2}{3}; 5]}$ $\quad$ 0,1 $\quad$ 0,2

0,2 $\quad$ $3x - 2 \leq 0 \Rightarrow x \leq \frac{2}{3} \Rightarrow \underline{f_4(x) = -3x + 2 \wedge]-\infty; \frac{2}{3}]}$ $\quad$ 0,1



II $\quad$ $f(0) = |-|-5| + 3| = |-2| = 2 \Rightarrow \underline{P_y(0|2)}$ $\quad$ 0,3

$f(x) = 0 \Rightarrow 2x + 3 = |x-5|$ $\quad$ 0,2

(auch aus dem Graphen) $\quad$ $\pm(2x+3) = x - 5$ $\quad$ $x_1 = -8 \notin \mathbb{L}$; $\quad$ $x_2 = \frac{2}{3} \Rightarrow \underline{P_x(\frac{2}{3}|0)}$