1 (5P)

a)

Variablen x : Anzahl Häuser Typ A

y : Anzahl Häuser Typ B

0,5P

Zahlenbereich $x, y \in \mathbb{N}$

Nicht - Negativität $x, y \geq 0$

Einschränkende Bedingungen :

b) 1) $400x + 600y \leq 12'000$

2) $300'000x + 200'000y \leq 6'000'000$

3) $18'000x + 6'000y \leq 324'000$

0,8P

4) **Zielfunktion** $z = 5x + 7y$

$z \rightarrow$ Maximal

$z_0 \Rightarrow \underline{y = \frac{-5x}{7}}$ 0,2P

c) 1) $y \leq -\frac{2x}{3} + 20$

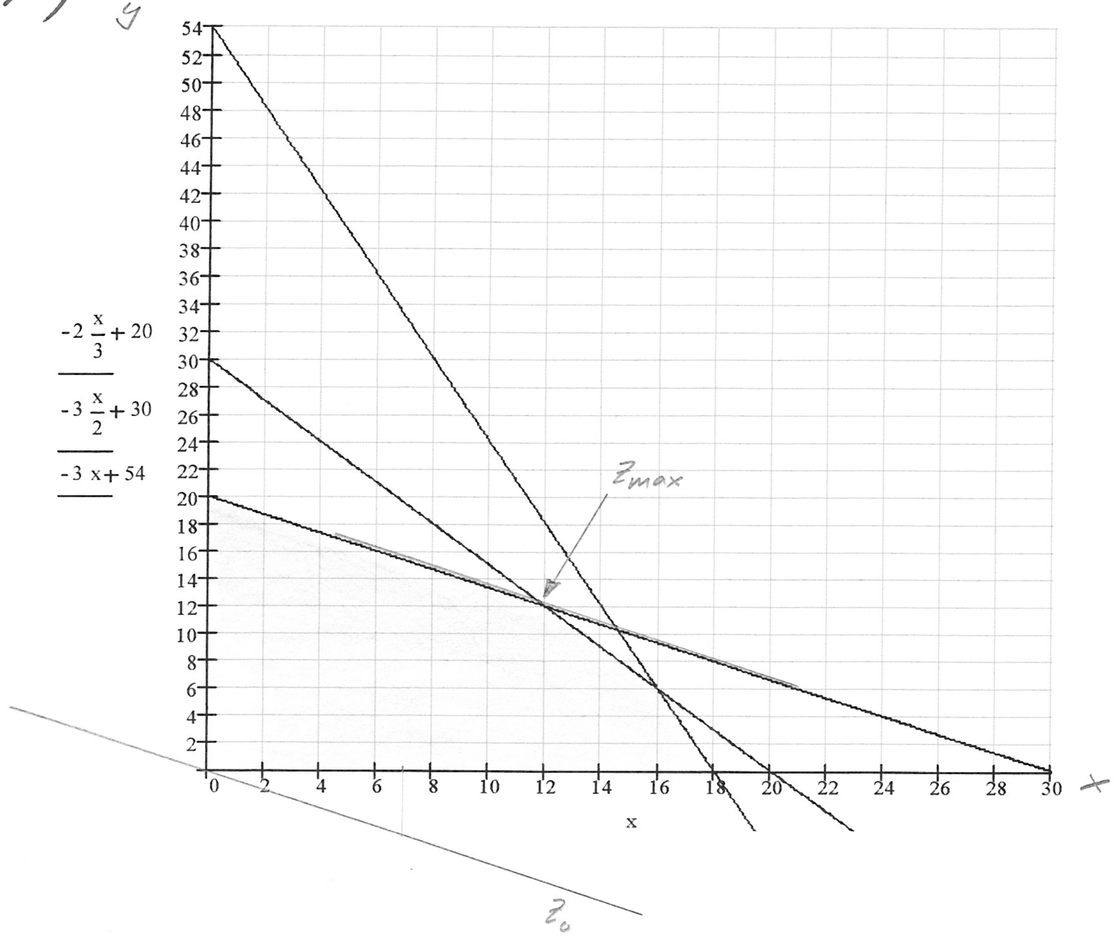
2) $y \leq -\frac{3x}{2} + 30$

3) $y \leq -3x + 54$

x_0	y_0
30	20
20	30
18	54

1,2P

1 (5P)



1,5P

d)

<u>Lösung aus dem Graphen</u> P (12/12)

0,4P

12 Häuser vom Typ A
und 12 Häuser vom Typ B

$z = 5 \cdot 12 + 7 \cdot 12 = \underline{\underline{144 \text{ Personen}}}$

0,4P

<u>Rechnerisch:</u>

$$-\frac{2x}{3} + 20 = -\frac{3x}{2} + 30$$

$$-\frac{2x}{3} + \frac{3x}{2} = 10$$

$$\frac{5x}{6} = 10$$

$$\underline{x = 12}$$

$$\Rightarrow \underline{y = 12}$$

• proportionale Abzüge

2 (4P)

A(1/1/-1)

B(3/3/2)

C(3/-1/-2)

a) Koordinatengleichung

$E: \ \vec{n} \cdot \vec{r} - \vec{n} \cdot \vec{r}_P = 0$

$\vec{r}_P = \vec{r}_A = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

$\vec{n} = \vec{AB} \times \vec{AC}$

$\vec{n} = \vec{AB} \times \vec{CA}$ 0,4P

$= \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & -2 \\ 3 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & -2 \\ 2 & 2 \end{vmatrix} \end{pmatrix}$

$\vec{n} = \begin{pmatrix} -4 \\ -8 \\ 8 \end{pmatrix} \Rightarrow \underline{\vec{n} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}}$ 0,6P

$\underline{\vec{AB}} = \vec{r}_B - \vec{r}_A$

$= \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \underline{\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}}$ 0,3P

$\underline{\vec{AC}} = \vec{r}_C - \vec{r}_A$

$= \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \underline{\begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}}$ 0,3P

$\vec{CA} = \begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$

$E: \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \vec{r} - \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 0$

$\underline{\underline{E: \ x + 2y - 2z - 5 = 0}}$ 0,4P

b) $\underline{\underline{\vec{n}_0}} = \frac{\vec{n}}{n} = \frac{1}{\sqrt{9}} \cdot \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \underline{\underline{\frac{1}{3} \cdot \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}}}$ 0,5P

2 (4P)

c) $$d = \frac{|\vec{n} \cdot (\vec{r}_Q - \vec{r}_1)|}{|\vec{n}|}$$

$$\Rightarrow \underline{d} = \frac{|-\vec{n} \cdot \vec{r}_1|}{n} = \underline{\frac{5}{3}} \text{ LE}$$ 0,5P

$$\vec{r}_F = \vec{n}_0 \cdot d = \begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix} \cdot \frac{5}{3}$$

$$\vec{r}_F = \begin{pmatrix} 5/9 \\ 10/9 \\ -10/9 \end{pmatrix} \Rightarrow \underline{F\left(\frac{5}{9} \,\Big|\, \frac{10}{9} \,\Big|\, -\frac{10}{9}\right)}$$ 0,5P

d) xy-Ebene:

E:

$$xy: \; \vec{n}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \vec{n} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

$$\cos\alpha = \frac{\vec{n}_1 \cdot \vec{n}}{n_1 \cdot n} = \frac{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}}{1 \cdot 3} = \frac{-2}{3}$$

0,7P

$$\underline{\alpha = 131,81°} \Rightarrow \underline{\alpha' = 48,19°}$$

• proportionale Abgriffe

3/3P)

$f(x) = 0{,}25x^2 + 2x - 5$

$g(x) = \frac{2x}{3} + t$

$f(x) \cap g(x)$

$$0{,}25x^2 + 2x - 5 = \frac{2x}{3} + t$$
$$0{,}75x^2 + 6x - 15 = 2x + 3t$$
$$0{,}75x^2 + 4x - 15 - 3t = 0$$ 1P

$D = 0$ $\quad D = b^2 - 4ac$

$$\Rightarrow 4^2 - 4 \cdot 0{,}75(-15 - 3t) = 0$$
$$4^2 + 45 + 9t = 0$$
$$t = \frac{-61}{9}$$

$$\Rightarrow g(x) = \frac{2x}{3} - \frac{61}{9}$$ 1P

$D = 0 \Rightarrow x_1 = x_2 = \frac{-b}{2a} = \frac{-4}{1{,}5} = \frac{-8}{3}$ 0,5P

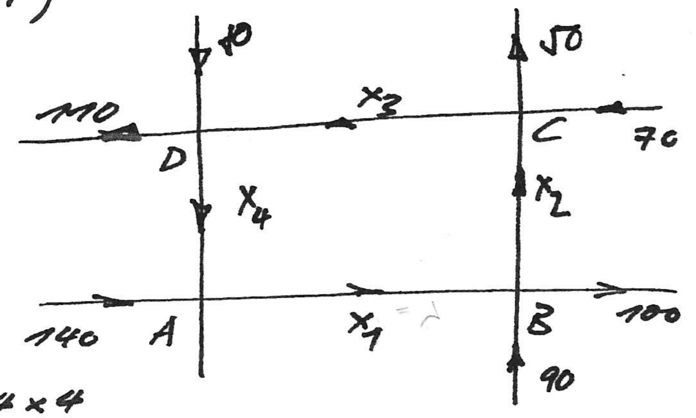
$$g\left(\frac{-8}{3}\right) = \frac{-16}{9} - \frac{61}{9} = \frac{-77}{9}$$

$$\Rightarrow T\left(\frac{-8}{3} \middle| \frac{-77}{9}\right)$$ 0,5P

· proportionale Abzüge

4 (3P)

a)



A: $x_1 + 120 = 140 + x_4$

B: $x_1 + 90 = 100 + x_2$

C: $x_2 + 70 = 50 + x_3$

D: $x_3 + 80 = 110 + x_4$

0,8P

4 × 4

$$\Rightarrow \left| \begin{array}{l} x_1 - x_4 = 20 \\ x_1 - x_2 = 10 \\ -x_2 + x_3 = 20 \\ x_3 - x_4 = 30 \end{array} \right| \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 20 \\ 1 & -1 & 0 & 0 & 10 \\ 0 & -1 & 1 & 0 & 20 \\ 0 & 0 & 1 & -1 & 30 \end{array} \right)]-$$

0,4P

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 20 \\ 0 & +1 & 0 & -1 & 10 \\ 0 & -1 & 1 & 0 & 20 \\ 0 & 0 & 1 & -1 & 30 \end{array} \right)]+ \left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 20 \\ 0 & 1 & 0 & -1 & 10 \\ 0 & 0 & 1 & -1 & 30 \\ 0 & 0 & 1 & -1 & 30 \end{array} \right) \Big\} \text{Identität}$$

0,3P 0,3P

$\Rightarrow \quad x_3 - x_4 = 30 \qquad \underline{\underline{x_3 = \lambda}}$ (Wegen Aufgabe b) ist hier x_3 als Parameter definiert)

0,2P

$\Rightarrow \underline{\underline{x_3 = \lambda}}$

$\underline{\underline{x_4 = \lambda - 30}}$

$\underline{x_2} = \lambda - 30 + 10 = \underline{\underline{\lambda - 20}}$

$\underline{x_1} = \lambda - 30 + 20 = \underline{\underline{\lambda - 10}}$

0,6P

b) $x_3 \leq 100$ A/h

$\Rightarrow x_4 \leq 70$ A/h

$x_2 \leq 80$ A/h

$\underline{\underline{x_1 \leq 90 \text{ A/h}}}$

0,4P

• proportionale Abgänge

5(3P)

$$3\tan(x) - \cos(x) = \frac{1}{\cos(x)} \quad , \; [-\pi, 2\pi]$$

$\underline{\mathbb{D}}$:

0,3P

$$\left.\begin{array}{ll} \tan(x): & \frac{-\pi}{2};\frac{\pi}{2};\frac{3\pi}{2} \\ \frac{1}{\cos(x)}: & \frac{-\pi}{2};\frac{\pi}{2};\frac{3\pi}{2} \end{array}\right\} \quad \mathbb{D} = [-\pi; 2\pi] \setminus \left\{\frac{-\pi}{2};\frac{\pi}{2};\frac{3\pi}{2}\right\}$$

0,2P

0,3P

$$\cos(x)\,3\tan(x) - \cos^2(x) = 1$$

$$3\,\frac{\sin(x)}{\cancel{\cos(x)}}\cdot\cancel{\cos(x)} - \cos^2(x) = 1$$

$$3\sin(x) - (1 - \sin^2(x)) = 1$$

$$3\sin(x) - 1 + \sin^2(x) = 1$$

$$\sin^2(x) + 3\sin(x) - 2 = 0$$ 0,8P

Substitution

$\sin(x) = a$

0,2P

$$a^2 + 3a - 2 = 0$$

$$a_{1/2} = \frac{-3 \pm \sqrt{9 + 4\cdot 1\cdot 2}}{2} \quad \begin{array}{l} a_1 = 0{,}5616 \\ a_2 = -3{,}5616 \;\notin \mathbb{R} \end{array}$$

0,6P

$\rightarrow \sin(x_0) = 0{,}5616$

$x_0 = 0{,}5963$ (0,2)

0,6P

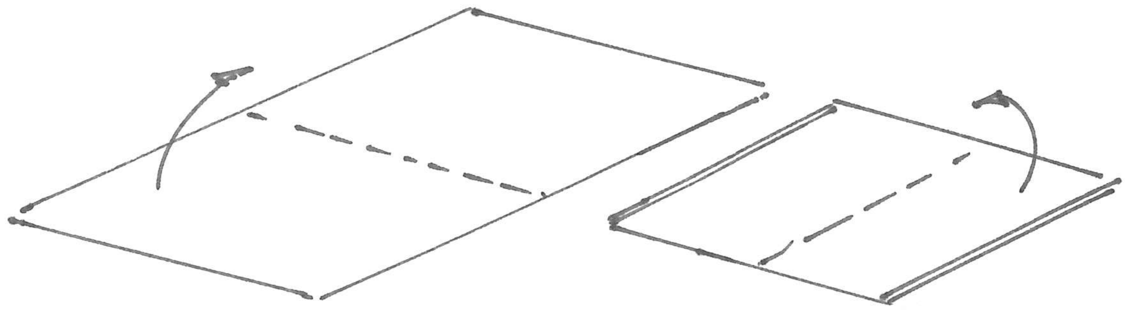
Periodizität:

$x = \pi - x_0 = 2{,}5453$ (0,2)

$\rightarrow \mathbb{L} = \{0{,}5963; 2{,}5453\}$ (0,2)

· proportionale Abzüge

6 (2P)



$f(x) = a \cdot b^x$ 0,4P

b : Wachstumsfaktor ⇒ Verdoppelung ⇒ $b=2$ 0,4P

a : Papierdicke ⇒ Gewichtung 0,2P

$\Rightarrow 55{,}758 \cdot 10^9 = 0{,}2 \cdot 10^{-3} \cdot 2^x$ 0,4P

$$2^x = 2{,}7879 \cdot 10^{14}$$

$$x = \frac{\ln(2{,}7879 \cdot 10^{14})}{\ln 2}$$

$x = 47{,}986$ ⇒ $\underline{\underline{48}}$ mal falten!

0,6P

• proportionale Abzüge

7 (4P)

a) $$\frac{3x}{x+5} \le \frac{6}{x-1} \qquad D = \mathbb{R} \setminus \{-5; 1\}$$

$$\frac{3x}{x+5} - \frac{6}{x-1} \le 0 \qquad | : 3$$

$$\frac{x}{x+5} - \frac{2}{x-1} \le 0$$

· prop. Abzüge

$$\frac{x^2 - x - 2x - 10}{(x+5)(x-1)} \le 0$$

$$\frac{x^2 - 3x - 10}{(x+5)(x-1)} \le 0$$

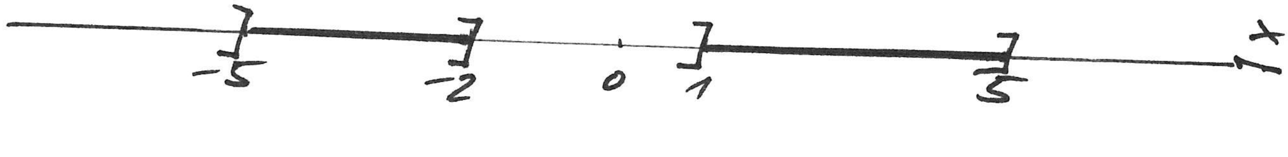
$$\Rightarrow \frac{(x+2)(x-5)}{(x+5)(x-1)} \le 0 \qquad 1P$$

Grenzwerte: $Z = 0 \Rightarrow x = -2$

$\wedge\ x = 5$ 0,4P

$N = 0 \Rightarrow x = -5$

Zahlenstrahl $x = 1$



Test mit $x = 0$ $\Rightarrow \frac{2 \cdot (-5)}{5(-1)} > 0\,!$ 0,2P

$$\Rightarrow \mathbb{L} = \left]-5; -2\right] \cup \left]1; 5\right]$$ 0,4P

7 (4P)

b) $4^{3x-1} \cdot 2^{4x} - 16 = 32^{x-0,2}$

$2^{6x-2} \cdot 2^{4x} - 16 = 2^{5x-1}$

$2^{10x-2} - 2^{5x-1} - 16 = 0$

$\left(2^{5x-1}\right)^2 - 2^{5x-1} - 16 = 0$ 0,6P

Substitution $a = 2^{5x-1}$ 0,2P

$\Rightarrow a^2 - a - 16 = 0$

$a_{1/2} = \dfrac{1 \pm \sqrt{1 + 4 \cdot 1 \cdot 16}}{2}$ 0,4P

$a_1 = 4,5311$ 0,4P

$a_2 = -3,5311 \notin \mathbb{L}$!

$\Rightarrow 2^{5x-1} = 4,5311 \quad | \ln$

$5x-1 = \dfrac{\ln 4,5311}{\ln 2}$

$5x-1 = 2,18$ 0,4P

$\underline{\underline{x}} = \dfrac{3,18}{5} = \underline{\underline{0,636}}$

· prop. Angabe

P (3P)

$$\frac{x^2-8x+4b}{x-3}-\frac{3b+20}{x-4}=\frac{x^2-9x-3b}{x-4}+\frac{b-6}{x-3}$$

$$\frac{x^2-8x+4b-b+6}{x-3}=\frac{x^2-9x-\cancel{3b}+\cancel{3b}+20}{x-4}$$

$$(x^2-8x+3b+6)(x-4)=(x^2-9x+20)(x-3)$$

$$\cancel{x^3}-\cancel{4x^2}-\cancel{8x^2}+32x+3bx-12b+6x-24$$

$$=\cancel{x^3}-\cancel{3x^2}-\cancel{9x^2}+27x+20x-60$$

$$3bx-12b=9x-36$$

$$3b(\cancel{x-4})=9(\cancel{x-4})$$

$$\underline{\underline{b=3}}$$

2,4P

Verschiedene Fehler 0,4P Abzug

<u>Verifizierung</u>

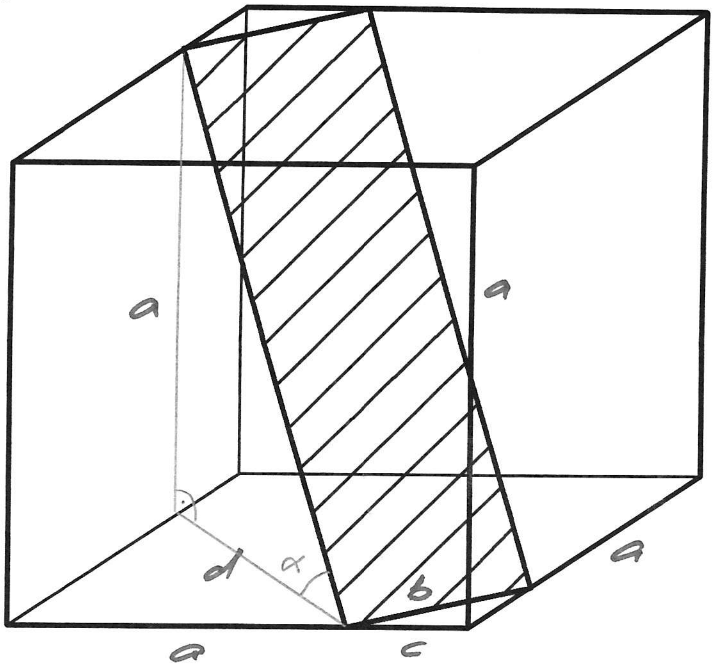
$$\frac{x^2-8x+12}{x-3}-\frac{9+20}{x-4}=\frac{x^2-9x-9}{x-4}+\frac{-3}{x-3}$$

$$\frac{x^2-8x+15}{x-3}=\frac{x^2-9x+20}{x-4}$$

0,6P

$$\frac{(\cancel{x-3})(x-5)}{(\cancel{x-3})}=\frac{(\cancel{x-4})(x-5)}{(\cancel{x-4})}\rightarrow \underline{\underline{0x=0}}$$

9 (3P)



a) $\tan\alpha = \dfrac{a}{d}$ $\Rightarrow$ $c = \dfrac{b}{\sqrt{2}}$ 0,4P

0,4P

$d = (a-c)\cdot\sqrt{2}$

$d = \left(a - \dfrac{b}{\sqrt{2}}\right)\sqrt{2}$

$\tan\alpha = \dfrac{a}{a\sqrt{2}-b}$ $\Leftarrow$ $d = (a\sqrt{2} - b)$ 0,6P

$\tan\alpha = \dfrac{10}{(10\sqrt{2} - 3)} = 0{,}897$

$\underline{\underline{\alpha = 41{,}9^\circ}}$ 0,6P

b) $\tan\alpha = \dfrac{a}{a\sqrt{2} - b}$

$\tan 45 = 1$ $\Rightarrow$ $1 = \dfrac{a}{a\sqrt{2} - b}$ 1P

• prop. Abzüge

$\underline{b} = a\sqrt{2} - a = \underline{\underline{a(\sqrt{2} - 1)}}$

10 (6P)

$f(x) = \ln(x) - 3 \qquad \mathbb{D} = \mathbb{R}^+$

$g(x) = -2x + 4 \qquad \mathbb{D} =]-\infty; 2[$

a) $h(x) = (f \circ g)(x)$

$\underline{\underline{h(x) = \ln(-2x+4) - 3}}$ 0,4P

b) $\underline{\underline{\mathbb{D}_h =]-\infty; 2[}}$; $\underline{\underline{W_h = \mathbb{R}}}$ 0,6P

c) <u>Nullstelle:</u>

$h(x_0) = 0$

$\ln(-2x_0+4) - 3 = 0$

$\ln(-2x_0+4) = 3 \qquad | e^{()}$

$-2x_0 + 4 = e^3$

$x_0 = \frac{1}{2}(4 - e^3)$

$\underline{\underline{x_0 = -8,043}}$ 1P

<u>Ordinatenabschnitt:</u>

$\underline{\underline{h(0)}} = \ln(4) - 3 = \underline{\underline{-1,614}}$ 0,4P

· prop. Abgriffe

10 (6P)

d) $y = \ln(-2x+4) - 3$

$y+3 = \ln(-2x+4)$

$e^{y+3} = -2x+4$

$x = \frac{1}{2}\left(4 - e^{y+3}\right)$

- nicht verlangt

$\underline{\underline{f^{-1}(x) = -\frac{1}{2}e^{x+3} + 2}}$

$\left(\underline{\underline{\mathbb{D}_{f^{-1}} = \mathbb{R}}} \quad \underline{\underline{\mathbb{W}_{f^{-1}} =]-\infty; 2[}} \right)$

e) $\underline{x_2 > x_1 \quad \wedge \quad x_2 \neq x_1}$!

Vorgabe

$\left(\text{streng monoton fallend: } \underline{\frac{f(x_2) - f(x_1)}{x_2 - x_1} < 0} \right)$

$\underline{x_2 - x_1 > 0}$ 0,2P

$f(x_2) - f(x_1) = -\frac{1}{2}e^{x_2+3} + \cancel{2} + \frac{1}{2}e^{x_1+3} - \cancel{2}$

$= \underline{\frac{1}{2}e^{3}\left(e^{x_1} - e^{x_2}\right)} \Rightarrow e^{x_1} < e^{x_2}$

$\Rightarrow \underline{e^{x_1} - e^{x_2} < 0}$

$\Rightarrow \underline{\underline{\frac{\frac{1}{2}e^{3}\left(e^{x_1} - e^{x_2}\right)}{x_2 - x_1} < 0}}$

0,8P

- Zahlenbeispiel auch möglich

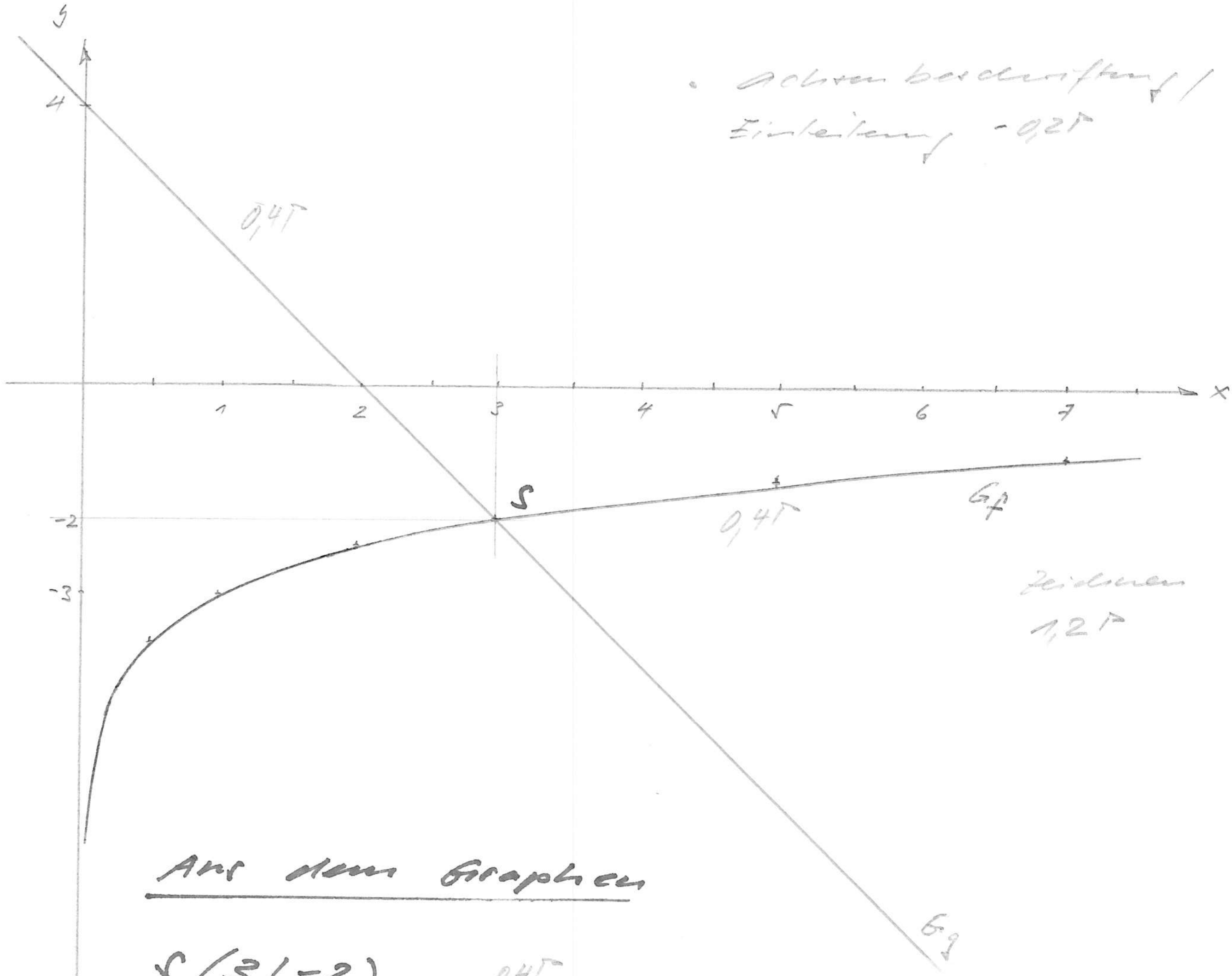
10 (6P)

f) $f(x) = \ln(x) - 3$ $g(x) = -2x + 4$

Wertetabelle

x	0	0,5	1	2	3	5	7	9	11
f(x)	$-\infty$	-3,693	-3	-2,307	-1,901	-1,391	-1,054	-0,8028	-0,6021
g(x)	4			0					

0,4P



• Achsenbeschriftung / Einteilung -0,2P

Zeichnen 1,2P

Aus dem Graphen

$S(3/-2)$ 0,4P

Lösung mit TR: $S(2{,}958/-1{,}916)$

11 (3P)

∘ prop. Abgriffe

$$g: \vec{r} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \qquad h: \vec{r} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

a) $E \perp yz$-Ebene $\wedge\ g \subset E$

$E: \vec{n} \cdot \vec{r} - \vec{n} \cdot \vec{r_p} = 0$ yz-Ebene:

$\vec{n} \perp \vec{n_1} \ \wedge\ \vec{n} \perp \vec{a}$ 0,25P $\vec{n_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ 0,25P

$\Rightarrow \vec{n} = \vec{n_1} \times \vec{a}$ $\vec{a} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ 0,25P

$$\underline{\vec{n}} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} 0 & 2 \\ 0 & -1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 4 \\ 0 & -1 \end{vmatrix} \\ \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} \end{pmatrix} = \underline{\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}}$$ 0,25P

$$E: \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \cdot \vec{r} - \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} = 0$$

$$\underline{\underline{E: y + 2z - 2 = 0}}$$ 0,5P

b) Gegenseitige Lage:

Parallelität: $\vec{a} = r\vec{b} \Rightarrow \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} = r \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$

$\Rightarrow r \neq$ konst.!

$\Rightarrow g \nparallel h$ 0,3P

Windschief:

$D(\vec{a}, \vec{b}, \vec{r_1} - \vec{r_2}) \Rightarrow \vec{r_1} - \vec{r_2} = \begin{pmatrix} 3-3 \\ 2-4 \\ 0-5 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ -5 \end{pmatrix}$ 0,3P

0,6P

$$D = \begin{vmatrix} 4 & 1 & 0 \\ 2 & 3 & -2 \\ -1 & -1 & -5 \end{vmatrix} \begin{matrix} 4 & 1 \\ 2 & 3 \\ -1 & -1 \end{matrix} = -60 + 2 + 0 - 0 - 8 + 10 = -56$$

$\Rightarrow D \neq 0 \Rightarrow$ $\underline{\underline{g + h \text{ sind windschief}}}$ 0,3P