1a

$$\frac{x-2}{|7-x|} \le \frac{1}{3}$$

$$\underline{\underline{D = \mathbb{R} \setminus \{7\}}}$$ 0,3P

$$\Rightarrow \quad x-2 \le \frac{1}{3}|7-x| \qquad |\cdot 3$$

$$3x-6 \le |7-x|$$ 0,5P

pos. Fall

$$7-x \ge 0$$

$$\underline{x \le 7}$$ 0,2P

$$\Rightarrow \quad 3x-6 \le 7-x$$

$$4x \le 13$$ 0,5P

$$\underline{x \le \frac{13}{4}} \quad \Rightarrow \quad \underline{x_1 \in \left]-\infty; \frac{13}{4}\right]}$$ 0,3P

neg. Fall

$$\underline{x > 7}$$ 0,2P

$$\Rightarrow \quad 3x-6 \le -(7-x)$$

$$3x-6 \le -7+x$$

$$2x \le -1$$ 0,5P

$$\underline{x \le -\frac{1}{2}} \quad \Rightarrow \quad \underline{x_2 \in \{\}}$$ 0,3P

$$\underline{\underline{L}} = L_1 \cup L_2 = \underline{\underline{\left]-\infty; \frac{13}{4}\right]}}$$ 0,2P

· abzüge proportional

1b

$$2ax - 24 = (x-3)(a+4)$$

$$2ax - 24 = ax + 4x - 3a - 12$$

$$ax - 4x = 12 - 3a$$

$$x(a-4) = -3(a-4) \qquad 1P$$

1. Fall $\quad a = 4 \Rightarrow 0x = 0 \Rightarrow \underline{\underline{L = \mathbb{R}}}$ $\quad 0{,}5P$

2. Fall $\quad a \neq 4 \Rightarrow \underline{\underline{x = -3}}$ $\quad 0{,}5P$

· prop. Abzüge

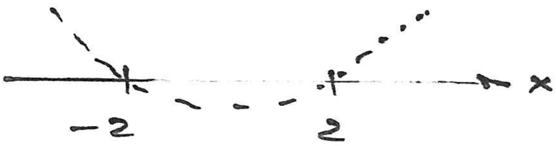
1c

$$\underset{①}{\log_{\sqrt{2}}(x-2)} = -3 + \underset{②}{\log_2(x^2-4)}$$

① $x-2 > 0$

$x > 2$ 0,2P

② $x^2-4>0$ $\quad x < -2 \ \vee\ x > 2$ 0,4P



$$\Rightarrow \underline{\underline{D = \,]2; \infty[}}$$

0,2P

$$\frac{\log_2(x-2)}{\frac{1}{2}\log_2 2} = -3 + \log_2(x^2-4)$$

$$\log_2(x-2)^2 - \log_2(x^2-4) = -3$$

$$\log_2\frac{(x-2)^2}{(x^2-4)} = -3 \qquad /2^{()}$$

$$\frac{\cancel{(x-2)}(x-2)}{\cancel{(x-2)}(x+2)} = \frac{1}{8}$$

$$x-2 = \frac{1}{8}(x+2)$$

$$8x-16 = x+2$$

$$7x = 18$$

1,2P

$$\underline{\underline{x = \frac{18}{7}}} \qquad \underline{\underline{L = \left\{\frac{18}{7}\right\}}}$$

- prop. Abzüge
- Fehler in der Ausrechnung 0,4P

2 a)

$$\lambda = x_5 + 100 \qquad \Rightarrow \qquad \underline{\underline{x_5 = \lambda - 100}}$$

$$x_5 + 110 = x_1 \qquad x_1 = \lambda - 100 + 110$$

$$x_1 = 100 + x_2 \qquad \underline{\underline{x_1 = \lambda + 10}}$$

$$x_2 + 80 = x_3 \qquad x_2 = x_1 - 100 = \lambda + 10 - 100$$

$$x_3 = x_4 + 60 \qquad \underline{\underline{x_2 = \lambda - 90}}$$

$$x_4 + 70 = \lambda$$

$$x_3 = \lambda - 90 + 80$$

$$\underline{\underline{x_3 = \lambda - 10}}$$

$$\underline{\underline{x_4 = \lambda - 70}}$$

(5 × 0,4P)

2P

b) Verkehrsfluss

max: $\underline{x_i = 130 \text{ Autos/h}}$

$$\Rightarrow \quad x_1 = 130 \text{ Autos/h} \quad \Rightarrow \quad \underline{\underline{\lambda = 120 \text{ Autos/h}}}$$

1P

$$\left(\begin{array}{l} x_5 = 20 \text{ Autos/h} \\ x_2 = 30 \text{ Autos/h} \\ x_3 = 110 \text{ Autos/h} \\ x_4 = 50 \text{ Autos/h} \end{array} \right)$$

• prop. abzüge

8

a) $p(x) = a(x+u)^2 + v$

$V(100/-2) \rightarrow p(x) = a(x-100)^2 - 2$ 0,6P

A: $p(0) = -42$

$-42 = 10'000\,a - 2$

$\underline{a = -0{,}004}$ 0,6P 0,8P

$\rightarrow \underline{\underline{p(x) = -0{,}004(x-100)^2 - 2}}$

b) $g(x) = mx + q$

$\underline{m} = \frac{\Delta g}{\Delta x} = \frac{0+42}{150-143} = \underline{6}$ 0,4P

$g(150) = 0 \Rightarrow \underline{q} = -6 \cdot 150 = \underline{-900}$ 0,4P

$\underline{\underline{g(x) = 6x - 900}}$ 0,2P

c) $\underline{p(x) = g(x)}$!

$-0{,}004(x-100)^2 - 2 = 6x - 900$

Value-Table: $\underline{x = 148{,}12}$ 1P

$\underline{\underline{B(148{,}12/-11{,}26)}}$ $\underline{y = -11{,}26}$

d) $d(PQ) \Rightarrow p(35) = -0{,}004(35-100)^2 - 2$

$p = -18{,}9$ 0,5P

$\Rightarrow \underline{\underline{d(PQ) = 18{,}9m}}$ $\rightarrow \underline{Q(35/-18{,}9)}$

4 a) $f_q(x) = 2x + q \quad , q \in \mathbb{R}$

$g(x) = mx + q \qquad$ Punkt $Q \in G_g$

$\Rightarrow m \cdot 2 = -1$ 0,5P

$m = \frac{-1}{2} \Rightarrow g(x) = -\frac{x}{2} + q$

$\Rightarrow Q: \quad 5 = -\frac{1}{2} + q$

$q = 5,5 \Rightarrow \underline{\underline{g(x) = -0,5x + 5,5}}$

0,5P 0,5P

b) $h(x) = mx + 3$

$\Rightarrow h(x_0) = f_q(x_0) = 0\,!$

$\Rightarrow h(x_0) = m \cdot x_0 + 3 = 0$

$x_0 = \frac{-3}{m} \rightarrow g(x_0)$ 0,5P

$\rightarrow f_q(x_0) = 2 \cdot x_0 + q = 2\left(\frac{-3}{m}\right) + q = 0$ 0,5P

$\rightarrow \frac{-6}{m} + q = 0$ 0,5P

$\underline{\underline{m \cdot q = 6}}$

• prop. abzüglich [illegible]

5 $$R(t) = e^{-\left(\frac{t}{T}\right)^b}$$

$t = 3$, $b = 2{,}3$

$R = 97\% \hat{=} 0{,}97$

• Umformungen à 0,5P

$$\Rightarrow \quad R = e^{-\left(\frac{t}{T}\right)^b} \quad | \ln$$

$$\ln R = -\left(\frac{t}{T}\right)^b$$

$$\sqrt[b]{-\ln R} = \frac{t}{T} \rightarrow \underline{\underline{T = \frac{t}{\sqrt[b]{-\ln R}}}}$$ 0,5P

$$T = \frac{3}{(-\ln 0{,}97)^{\frac{1}{2{,}3}}}$$

$$\underline{\underline{T = 13{,}69\ a}}$$ 0,5P

3P

6a

$$f(x) = a \sin(\omega x + \varphi) + d$$

<u>Aus dem Graphen:</u>

<u>$d = 0$</u>

<u>$a = 1{,}5$</u> 0,5P

Periodenlänge: $\frac{P}{2} = \frac{\pi}{3}$

$$P = \frac{2\pi}{3} \rightarrow \omega = \frac{\cancel{2\pi}}{\cancel{2\pi}} \cdot 3$$

0,5P

<u>$\omega = 3$</u> 0,5P

<u>Phasenverschiebung:</u>

$$-\frac{\varphi}{\omega} = -\frac{\pi}{12} \rightarrow \varphi = -\omega \cdot \left(\frac{-\pi}{12}\right) = \frac{3\pi}{12} = \frac{\pi}{4}$$ 0,5P

$$\Rightarrow \underline{\underline{f(x) = 1{,}5 \sin\left(3x + \frac{\pi}{4}\right)}}$$ 0,5P

b) $\sqrt{2}\cos\left(\frac{\pi}{4} + x\right) = \cos(x) - \sin(x)$

$$\cos\left(\frac{\pi}{4} + x\right) = \underbrace{\cos\left(\frac{\pi}{4}\right)}_{\frac{\sqrt{2}}{2}}\cos(x) - \underbrace{\sin\left(\frac{\pi}{4}\right)}_{\frac{\sqrt{2}}{2}} \cdot \sin(x)$$

2P

$$\Rightarrow \sqrt{2}\left(\frac{\sqrt{2}}{2}\cos(x) - \frac{\sqrt{2}}{2}\sin(x)\right) = \underline{\underline{\cos(x) - \sin(x)}}$$

q.e.d

6c $3\cos^2(x) - 5\sin(x) = 1$ $\qquad D = \mathbb{R}$

$\cos^2(x) = 1 - \sin^2(x)$

$\Rightarrow 3(1 - \sin^2(x)) - 5\sin(x) = 1$

$3 - 3\sin^2(x) - 5\sin(x) = 1$

$\Rightarrow 3\sin^2(x) + 5\sin(x) - 2 = 0$ 0,5P

Substitution : $a = \sin(x)$

$\Rightarrow 3a^2 + 5a - 2 = 0$ 0,5P

$a_{1/2} = \frac{-5 \pm \sqrt{25 + 4\cdot 3\cdot 2}}{6}$

$a_1 = \frac{-5+7}{6} = \frac{1}{3}$

$a_2 = \frac{-5-7}{6} = -2$

$x_1 = \sin^{-1} a_1 = \sin^{-1}\left(\frac{1}{3}\right) = 0{,}34$

$x_2 \notin \mathbb{R}$! 0,5P

Periodizität

$x_n = x_1 + 2k\cdot\pi = 0{,}34 + 2k\cdot\pi \ , \ k \in \mathbb{Z}$ 0,5P

$x_m = (2k+1)\pi - x_1 = (2k+1)\pi - 0{,}34 \ , \ k \in \mathbb{Z}$ 0,5P

[illegible]

2,5P

7 a) - Menge des Vitaminpräparates $V_1 : x$
- Menge des Vitaminpräparates $V_2 : y$ 0,5P

b) Bedingungen

Nicht neg. $x, y \geq 0 \wedge x, y \in \mathbb{Q}^+$

A : $0{,}10x + 0{,}15y \geq 1{,}50 \Rightarrow y \geq \frac{-2x}{3} + 10$ 0,5P

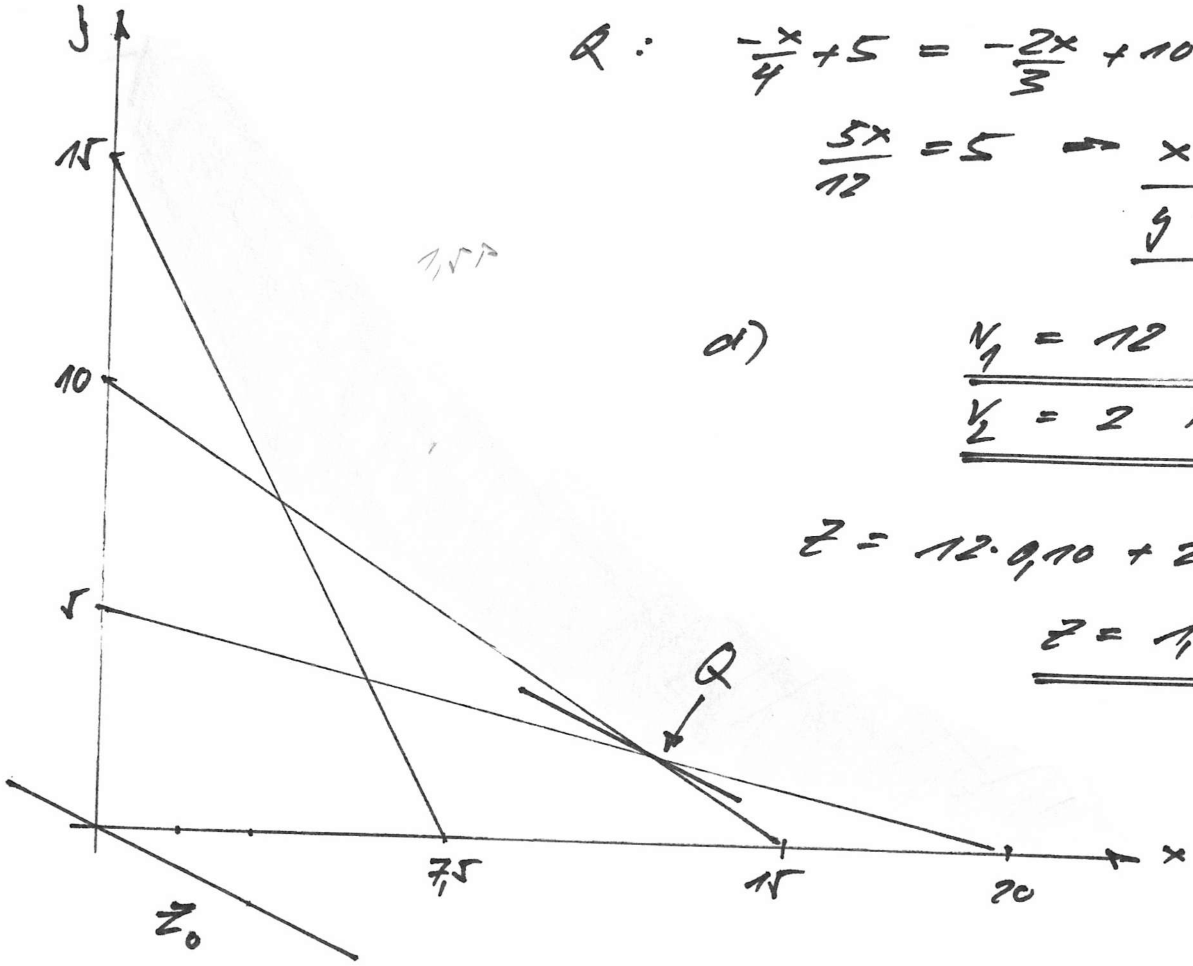
C : $20{,}00x + 10{,}00y \geq 150{,}00 \Rightarrow y \geq -2x + 15$ 0,5P

K : $1{,}00x + 4{,}00y \geq 20{,}00 \Rightarrow y \geq -\frac{x}{4} + 5$ 0,5P

$Z = 0{,}10x + 0{,}20y$

$\underline{y = -0{,}5x + 5Z}$ 0,5P

c)



1,5P

Q : $-\frac{x}{4} + 5 = -\frac{2x}{3} + 10$

$\frac{5x}{12} = 5 \Rightarrow \underline{x = 12}$

$\underline{y = 2}$

d) $\underline{\underline{V_1 = 12 \text{ Präp.}}}$

$\underline{\underline{V_2 = 2 \text{ Präp.}}}$ 0,5P

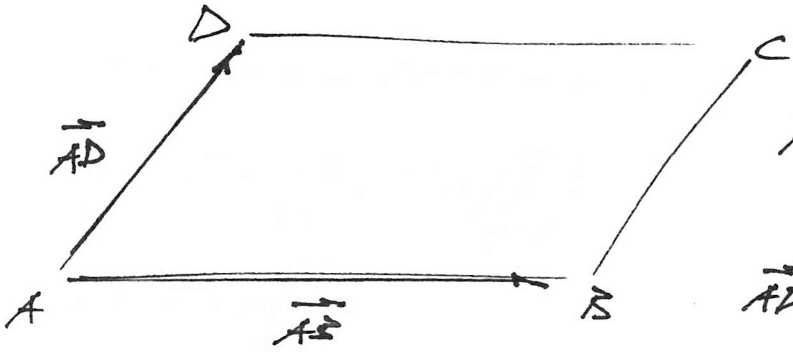
$Z = 12 \cdot 0{,}10 + 2 \cdot 0{,}20$

$\underline{\underline{Z = 1{,}60 \text{ GE}}}$ 0,5P

· prop. abzüge

5P

8a



$\vec{AB} = \begin{pmatrix} 0-2 \\ 7-4 \\ 7-5 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$ 0,2P

$\vec{AD} = \begin{pmatrix} -1-2 \\ 7-4 \\ 11-5 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 6 \end{pmatrix}$ 0,2P

$$E: \vec{r} = \begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} + s\begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix} + t\begin{pmatrix} -3 \\ 3 \\ 6 \end{pmatrix}$$ 0,6P

$$g: \vec{r} = v\begin{pmatrix} -2 \\ 15 \\ 20 \end{pmatrix}$$ 0,2P

$E \cap g = \{D\}$: $\begin{pmatrix} 2 \\ 4 \\ 5 \end{pmatrix} = s\begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix} + t\begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix} + v\begin{pmatrix} -2 \\ 15 \\ 20 \end{pmatrix}$

3x3
$$\left| \begin{array}{l} 2s + 3t - 2v = 2 \\ -3s - 3t + 15v = 4 \\ -2s - 6t + 20v = 5 \end{array} \right|$$ 0,6P

solve TI: $s = \frac{1}{2}$; $t = \frac{2}{3}$; $v = \frac{1}{2}$

Punkt E: $\vec{r}_E = \begin{pmatrix} -1 \\ 7,5 \\ 10 \end{pmatrix}$ 0,2P

- prop.

b) $s\,\vec{AB} \leq \vec{AB} \Rightarrow s \leq 1$ 1P

$t\,\vec{AD} \leq \vec{AD} \Rightarrow t \leq 1$

$\vec{AB}$ und $\vec{AD}$ definieren die Parallelogrammfläche; s, t sind kleiner 1, somit liegt der Punkt E innerhalb.

9

$E:\ x - y + 2z - 3 = 0$ · prop. abzüge

$g:\ \vec{r} = \begin{pmatrix}2\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}5\\3\\-1\end{pmatrix}$

$h:\ \vec{r} = \mu\begin{pmatrix}3\\2\\-2\end{pmatrix}$

a) $g \subset E \Rightarrow \vec{n}\cdot\vec{a} = 0$

$\wedge\ P \in E\ ;\ \vec{r}_P = \begin{pmatrix}2\\1\\1\end{pmatrix}$ 0,5P

$\Rightarrow E:\ \underline{2 - 1 + 2\cdot 1 - 3 = 0}$ ok.

0,5P $\vec{n} = \begin{pmatrix}1\\-1\\2\end{pmatrix}\ ;\ \vec{a} = \begin{pmatrix}5\\3\\-1\end{pmatrix}$

0,5P

$\Rightarrow \underline{\vec{n}\cdot\vec{a}} = \underline{\begin{pmatrix}1\\-1\\2\end{pmatrix}\begin{pmatrix}5\\3\\-1\end{pmatrix} = 5 - 3 - 2 = 0}$ ok.

1,5P

b) $G \perp xy\text{-Ebene} \wedge g \subset G$

$G:\ \vec{n}_G(\vec{r} - \vec{r}_P) = 0$

$\vec{r}_P = \begin{pmatrix}2\\1\\1\end{pmatrix}\ ;\ xy\text{-Ebene}:\ \vec{n}_{xy} = \vec{e}_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$ 0,5P

$\vec{n}_G = \vec{e}_3 \times \vec{a}_g = \begin{pmatrix}0\\0\\1\end{pmatrix} \times \begin{pmatrix}5\\3\\-1\end{pmatrix} = \begin{pmatrix}\begin{vmatrix}0 & 3\\1 & -1\end{vmatrix}\\ -\begin{vmatrix}0 & 5\\1 & -1\end{vmatrix}\\ \begin{vmatrix}0 & 5\\0 & 3\end{vmatrix}\end{pmatrix} = \begin{pmatrix}-3\\5\\0\end{pmatrix}$ 0,5P

$G:\ \underline{\underline{-3x + 5y + 1 = 0}}$ 0,5P

1,5P

c) $\begin{pmatrix}2\\1\\1\end{pmatrix} + \lambda\begin{pmatrix}5\\3\\-1\end{pmatrix} = \mu\begin{pmatrix}3\\2\\-2\end{pmatrix} \Rightarrow \underline{2\times 2}\ \begin{vmatrix}3\mu - 5\lambda = 2\\ 2\mu - 3\lambda = 1\end{vmatrix}\ \begin{matrix}1)\\2)\end{matrix}$

$\mu = \dfrac{2+5\lambda}{3} = \dfrac{1+3\lambda}{2}$

$4 + 10\lambda = 3 + 9\lambda$ 1P

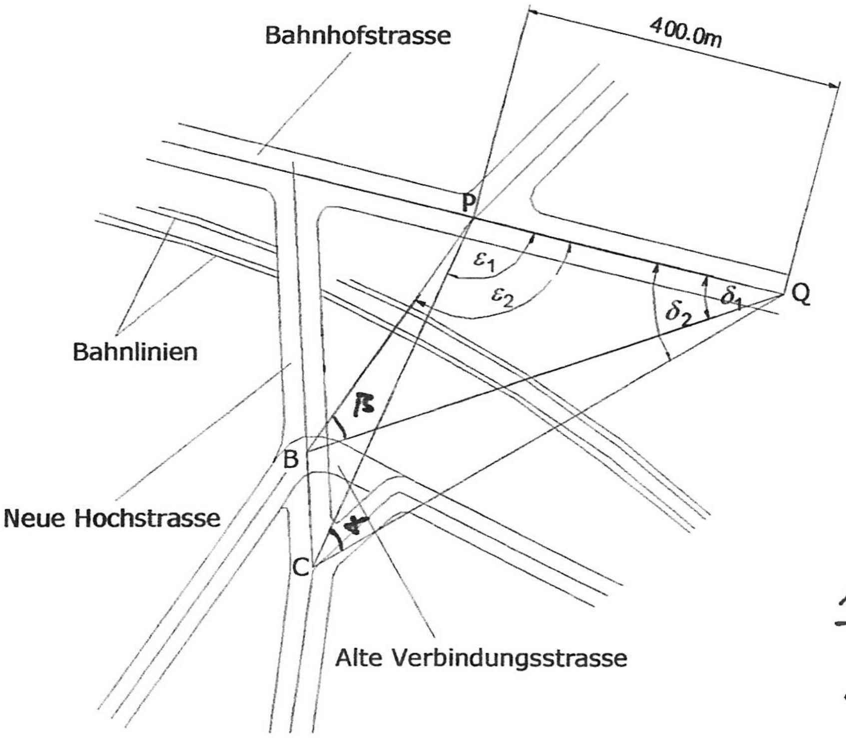
$\underline{\lambda = -1}\ ;\ \underline{\mu = -1}$

$\Rightarrow \underline{\underline{S}}:\ \vec{r}_S = \begin{pmatrix}2-5\\1-3\\1+1\end{pmatrix} = \underline{\underline{\begin{pmatrix}-3\\-2\\2\end{pmatrix}}}$ 0,5P

$\cos\varphi = \dfrac{\begin{pmatrix}5\\3\\-1\end{pmatrix}\begin{pmatrix}3\\2\\-2\end{pmatrix}}{\left|\begin{pmatrix}5\\3\\-1\end{pmatrix}\right|\left|\begin{pmatrix}3\\2\\-2\end{pmatrix}\right|} = 0{,}94$

$\underline{\underline{\varphi = 19{,}45^\circ}}$ 0,5P

10



$$\beta = 180 - (\varepsilon_2 + \delta_1)$$

$$\beta = 180 - (114 + 33{,}5)$$

$$\underline{\beta = 32{,}5^\circ}$$ 0,5P

$$\gamma = 180 - (\varepsilon_1 + \delta_2)$$

$$\gamma = 180 - (109{,}2 + 46{,}3)$$

$$\underline{\gamma = 24{,}5^\circ}$$ 0,5P

<u>Sinussatz:</u>

$$\frac{\overline{PC}}{\sin 46{,}3^\circ} = \frac{\overline{PQ}}{\sin 24{,}5^\circ}$$

$$\overline{PC} = \overline{PQ}\,\frac{\sin 46{,}3^\circ}{\sin 24{,}5^\circ} = 400 \cdot \frac{\sin 46{,}3^\circ}{\sin 24{,}5^\circ}$$

$$\underline{\underline{\overline{PC} = 697{,}35\,m}}$$ 1P

$$\overline{PB} = \overline{PQ} \cdot \frac{\sin 33{,}5^\circ}{\sin 32{,}5^\circ} = 400 \cdot \frac{\sin 33{,}5^\circ}{\sin 32{,}5^\circ}$$

$$\underline{\underline{\overline{PB} = 410{,}90\,m}}$$ 1P

<u>Cosinussatz:</u>

$$\overline{BC}^2 = \overline{PB}^2 + \overline{PC}^2 - 2\,\overline{PB}\,\overline{PC}\cos(114 - 109{,}2)$$

$$\overline{BC}^2 = 410{,}90^2 + 697{,}35^2 - 2 \cdot 410{,}90 \cdot 697{,}35 \cdot \cos 4{,}8^\circ$$

$$\underline{\underline{\overline{BC} = 289{,}94\,m}}$$ 1P

· prop. Abzüge