1a

$$\begin{vmatrix} ax - 2y = 1 \\ x - ay = a \end{vmatrix}$$

$$\underline{D} = \begin{vmatrix} a & -2 \\ 1 & -a \end{vmatrix} = -a^2 + 2 = -(a^2-2) = \underline{2-a^2}$$ 0,5P

$$\underline{Dx} = \begin{vmatrix} 1 & -2 \\ a & -a \end{vmatrix} = -a + 2a = \underline{a}$$ 0,5P

$$\underline{Dy} = \begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix} = \underline{a^2 - 1}$$ 0,5P

<u>Lösungsmenge:</u>

$$D \cdot x = Dx$$

$$(2-a^2)x = a \qquad | \cdot -1$$

$$(\sqrt{2}-a)(\sqrt{2}+a)x = a$$ 0,5P

<u>Fallunterscheidung:</u>

1. $\underline{\underline{a = \pm\sqrt{2}}} \Rightarrow 0x = \pm\sqrt{2} \rightarrow \underline{\underline{x \in \emptyset}}$ 1P

2. $\underline{\underline{a \neq \pm\sqrt{2}}} \rightarrow \underline{\underline{x}} = \underline{\underline{\frac{a}{2-a^2}}}$ 0,5P

$$\underline{\underline{y}} = \frac{Dy}{D} = \underline{\underline{\frac{a^2-1}{2-a^2}}}$$ 0,5P

· prop. abzüge

1b

$$x^2 + px + 4{,}5 = 0$$

Viëta:

$$\left|\begin{array}{lll} x_1 + x_2 = -p & & 0{,}5\,P \\ x_1 \cdot x_2 = 4{,}5 & & 0{,}5\,P \\ x_1 = 2x_2 & & 0{,}5\,P \end{array}\right.$$

$$\Rightarrow \quad 2x_2^2 = 4{,}5$$

$$x_2^2 = 2{,}25$$

$$|x_2| = \sqrt{2{,}25}$$

$$x_{21} = 1{,}5 \quad \Rightarrow \quad x_{11} = 3 \qquad 1\,P$$

$$x_{22} = -1{,}5 \quad \Rightarrow \quad x_{12} = -3$$

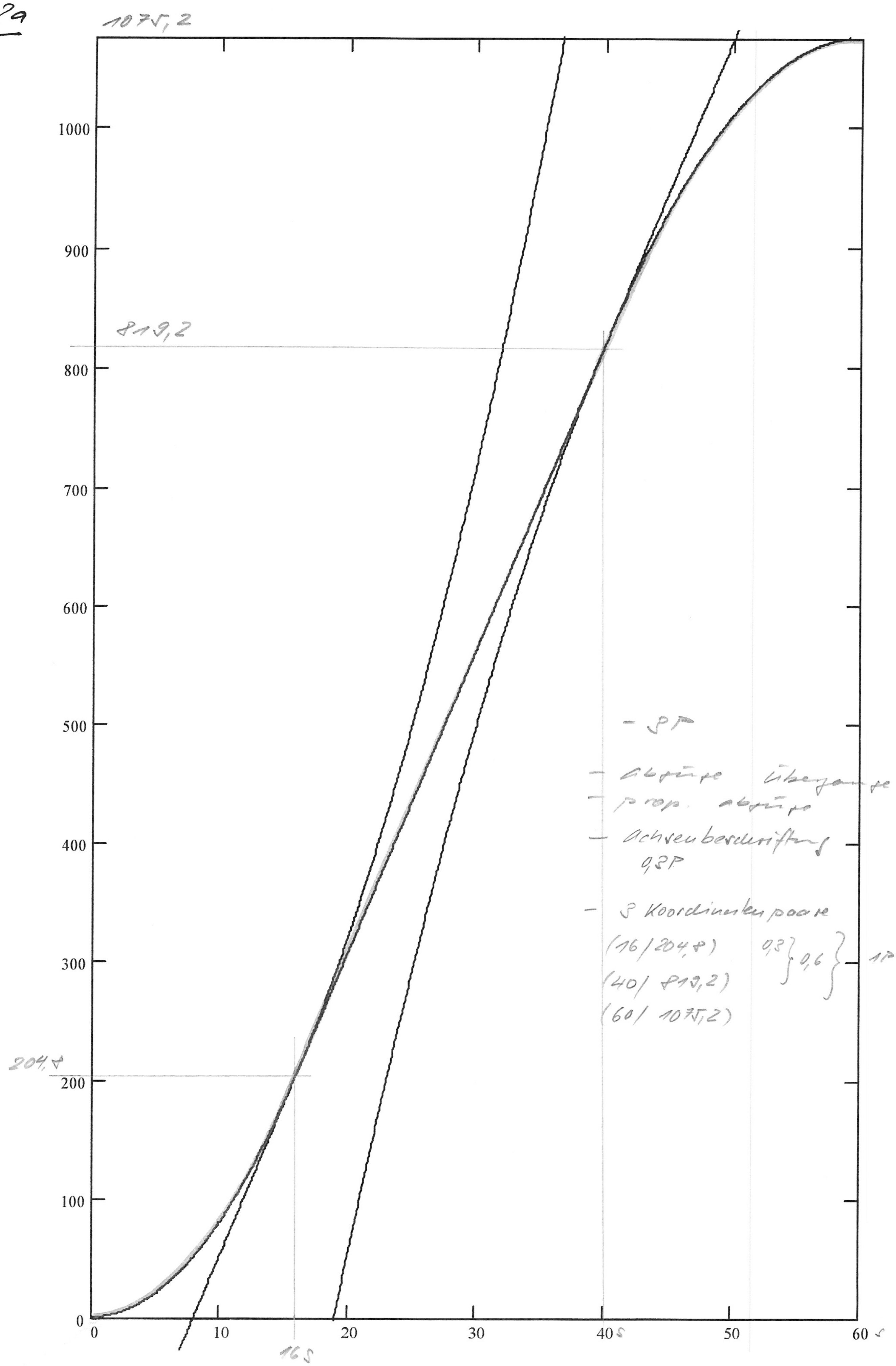
$$\underline{x_{22},\, x_{12} \notin \mathbb{N}}$$

$$\Rightarrow \quad x_1 = 3, \quad x_2 = 1{,}5$$

$$\underline{\underline{p}} = -(x_1 + x_2) = \underline{\underline{-4{,}5}} \qquad 0{,}5\,P$$

$$\Rightarrow \quad \underline{\underline{x^2 - 4{,}5x + 4{,}5 = 0}}$$

2a



- 3P

- abstürze übergänge } 0,3P
- prop. abstürze

- Achsenbeschriftung 0,3P

- 3 Koordinatenpaare
(16/204,8) 0,3 } 0,6 } 1P
(40/819,2)
(60/1075,2)

26

Der Haltestellenabstand kann aus Abschnitt (3) entnommen werden

(3) $\underline{\underline{s_3(60) = 1075{,}2\ m}}$ 1P

<u>Max. Geschwindigkeit</u>

→ im Diagramm = max. Steigung

Lösung im Abschnitt (2)

(2) $s_2(t) = 25{,}6\,(t-16) + 204{,}8$ 1P

$\Rightarrow m = 25{,}6 = v_{max}$

$\underline{\underline{v_{max} = 25{,}6\ m/s \;\hat{=}\; 92{,}2\ km/h}}$

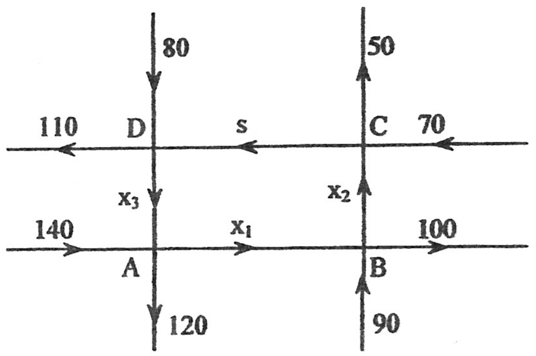
<u>Durchschnittsgeschwindigkeit</u>

1P

$\underline{v} = \frac{\Delta s}{\Delta t} = \frac{1075{,}2}{60} = 17{,}9\ m/s = \underline{\underline{64{,}5\ km/h}}$

3

a)



$D: \; s + 80 = 110 + x_3$

$C: \; x_2 + 70 = 50 + s$

$B: \; x_1 + 90 = 100 + x_2$

$A: \; 140 + x_3 = x_1 + 120$

1P

$A: \quad x_1 - x_3 = 20$

$B: \quad x_1 - x_2 = 10$

$C: \quad s - x_2 = 20 \quad \rightarrow \quad \underline{\underline{x_2}} = f(s) = \underline{\underline{s - 20}}$ 0,5P

$D: \quad s - x_3 = 30 \quad \rightarrow \quad \underline{\underline{x_3}} = f(s) = \underline{\underline{s - 30}}$ 0,5P

1P

$\rightarrow \; B: \quad x_1 - s + 20 = 10$

$\underline{\underline{x_1}} = f(s) = \underline{\underline{s - 10}}$ 0,5P

b) <u>max. Verkehrsfluss $\overline{AB}$</u> :

$\underline{\underline{x_1 = 30 \text{ Autos/h}}}$ 0,5P

$\rightarrow \quad \underline{\underline{s = 40 \text{ Autos/h}}} \quad \rightarrow \left(\begin{array}{l} x_2 = 20 \text{ Autos/h} \\ x_3 = 10 \text{ Autos/h} \end{array} \right)$

1P

· prop. Abzüge

4 Anzahl Mathematikbücher : x

Anzahl Physikbücher : y 0,5P

$x, y \in \mathbb{N}$, $x, y \geq 0$

Bedingungen

1) $x \geq 50$ } 0,5P

2) $y \geq 80$

3) $x + y \geq 200$ 0,5P

4) $32x + 40y \leq 9000 \Rightarrow y = \frac{-4x}{5} + 225$ 1P

5) $z = 5x + 6y$ (Min) $\rightarrow y = \frac{-5x}{6} + \frac{z}{6}$ 1P

Lösungen: $x = 120$, $y = 80$ 0,5P

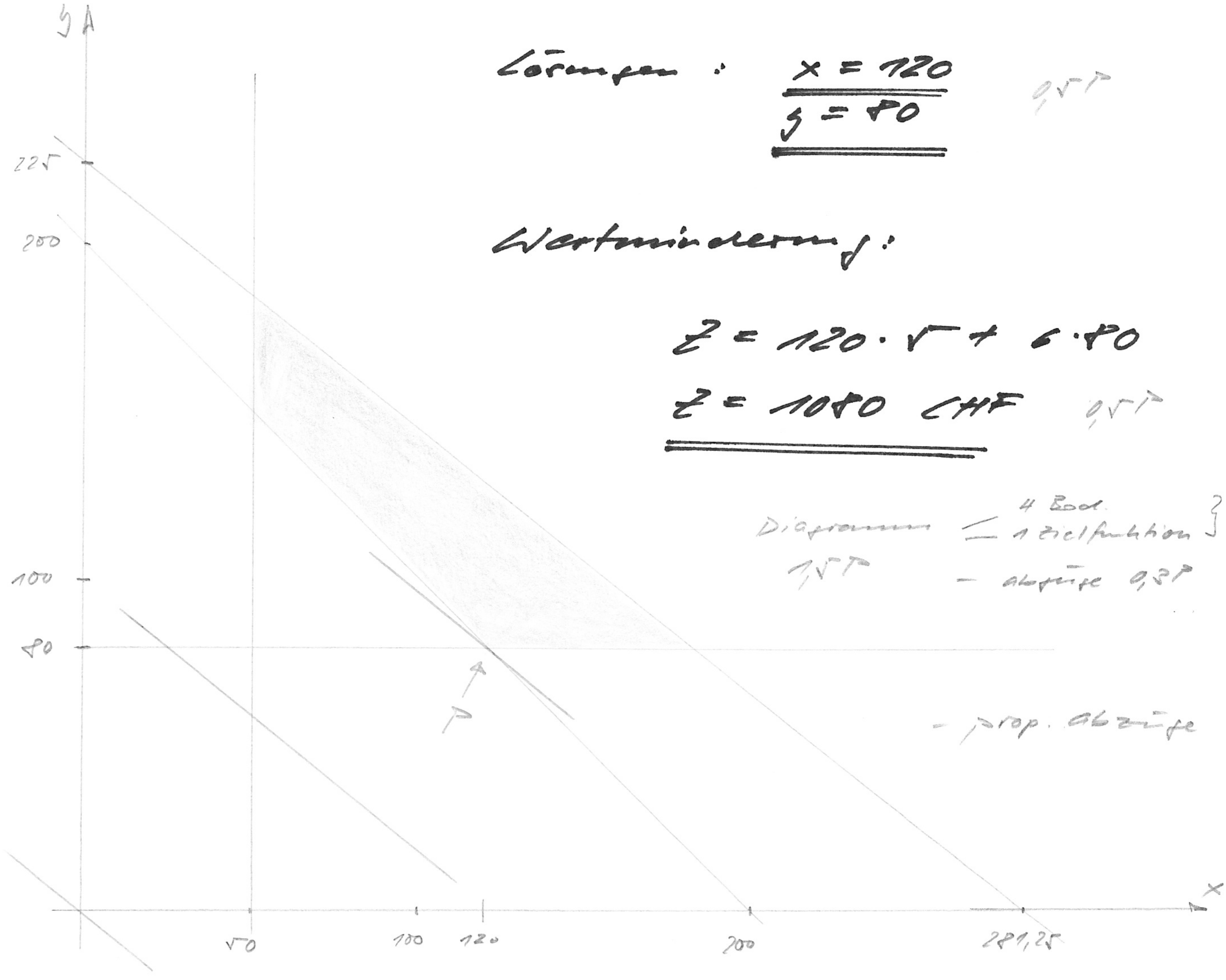
Wertminderung:

$z = 120 \cdot 5 + 6 \cdot 80$

$z = 1080$ CHF 0,5P

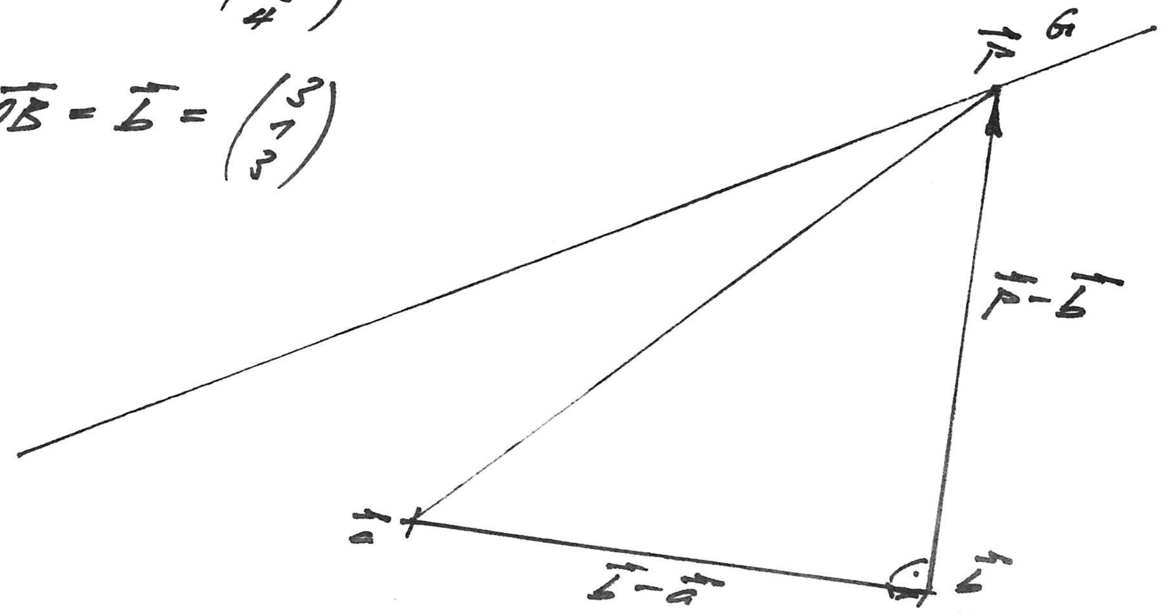
Diagramm 1,5P: 4 Bed., 1 Zielfunktion, je 0,5P; − Abzüge 0,5P

− prop. Abzüge



5 $\vec{OA} = \vec{a} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$

$\vec{OB} = \vec{b} = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$



$\vec{p} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$; $G: \vec{r} = \begin{pmatrix} -t \\ 2t \\ 3t \end{pmatrix} \rightarrow \vec{p} = \begin{pmatrix} -t \\ 2t \\ 3t \end{pmatrix}$ 0,5P

$(\vec{b}-\vec{a})(\vec{p}-\vec{b}) = 0$ → Skalarprodukt 0,5P

$\begin{pmatrix} 3-2 \\ 1-3 \\ 3-4 \end{pmatrix} \begin{pmatrix} -t-3 \\ 2t-1 \\ 3t-3 \end{pmatrix} = 0$

0,5P 0,5P

$\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \begin{pmatrix} -t-3 \\ 2t-1 \\ 3t-3 \end{pmatrix} = 0$

$-t-3-4t+2-3t+3 = 0$

$-8t = -2$ 0,5P

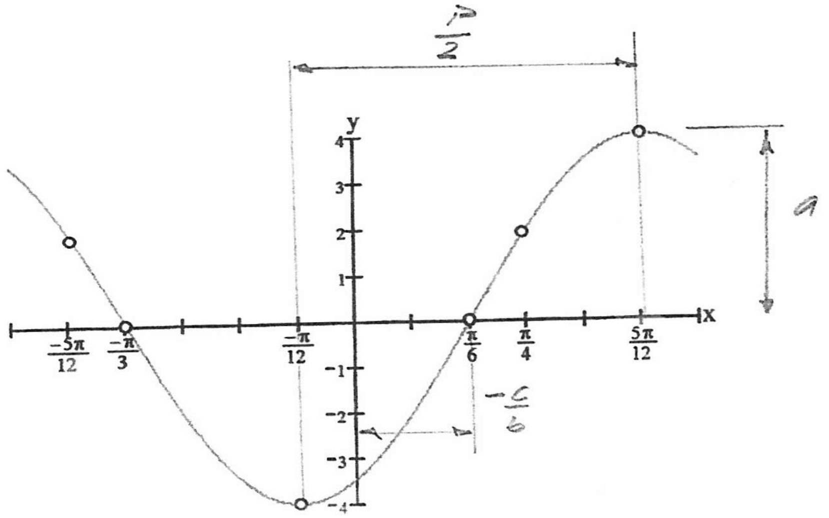
$t = \frac{1}{4}$

$P \rightarrow \vec{p} = \begin{pmatrix} -1/4 \\ 2/4 \\ 3/4 \end{pmatrix} = \begin{pmatrix} -1/4 \\ 1/2 \\ 3/4 \end{pmatrix}$ 0,5P

° prop. Abzüge

6

a)



allg. Sinusschwingung:

$$y = f(x) = a \cdot \sin(bx + c) + d$$

aus dem Graphen: $\left.\begin{array}{l} d = 0 \\ a = 4 \end{array}\right\}$ 0,5P

$$\frac{p}{2} = \frac{5\pi}{12} - \frac{-\pi}{12}$$

$$\frac{p}{2} = \frac{\pi}{2} \rightarrow p = \pi$$ 0,5P

$$\underline{b} = \frac{p_0}{p} = \frac{2\pi}{\pi} = \underline{2}$$ 0,5P

$$\frac{-c}{b} = \frac{\pi}{6}$$

$$\underline{c} = -b \cdot \frac{\pi}{6} = \underline{-\frac{\pi}{3}}$$ 0,5P

$$\rightarrow \underline{\underline{y = f(x) = 4 \cdot \sin\left(2x - \frac{\pi}{3}\right)}}$$ 0,5P

6b) $f(x) = -x^2$, $x \geq 0$

$\Rightarrow \underline{\underline{W_f =]-\infty ; 0]}}$

$f^{-1}(x)$:

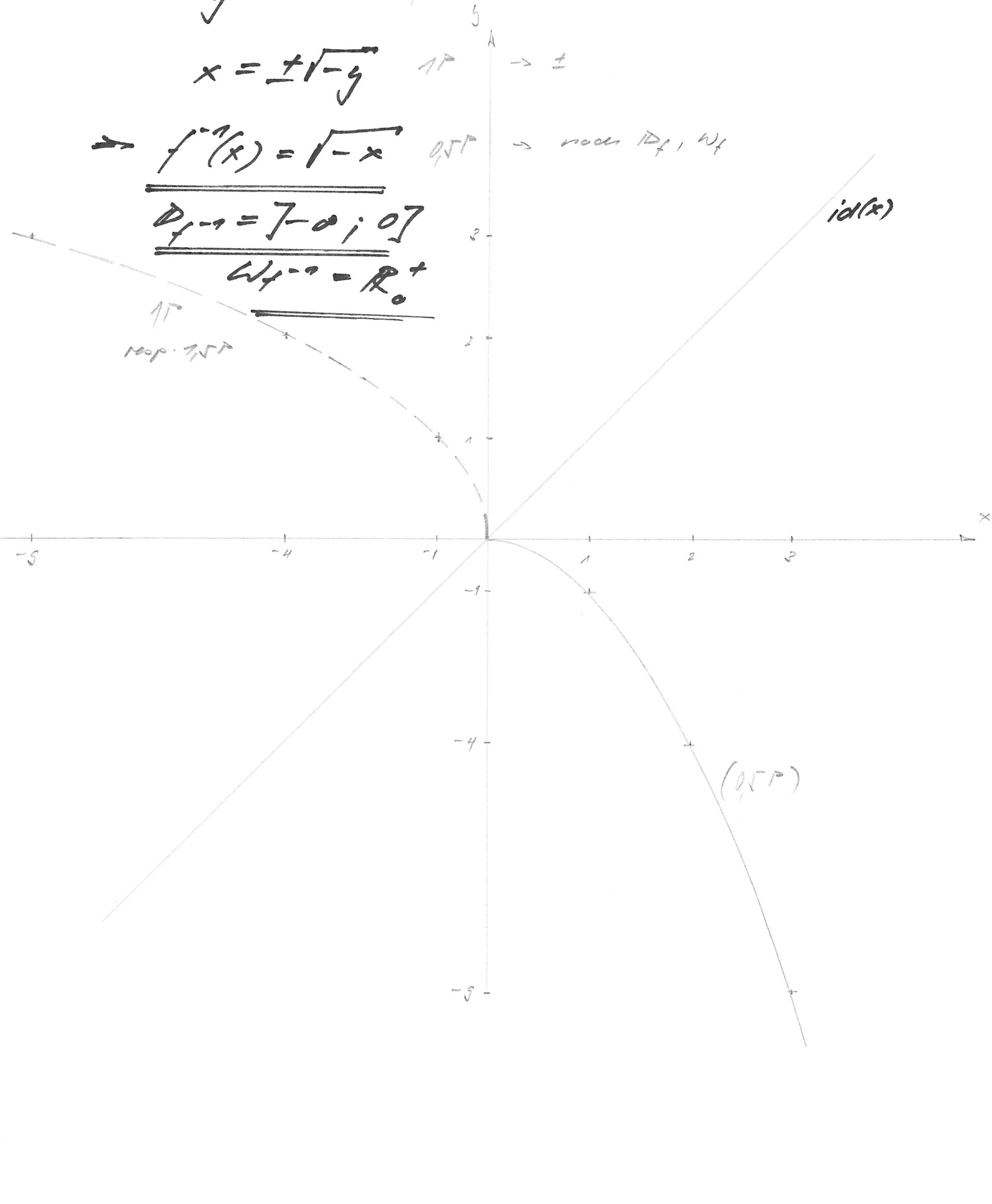
$-y = x^2$ 0,5P

$x = \pm\sqrt{-y}$ 1P → ±

$\Rightarrow \underline{\underline{f^{-1}(x) = \sqrt{-x}}}$ 0,5P → nach D_f, W_f

$\underline{\underline{D_{f^{-1}} =]-\infty ; 0]}}$

$\underline{\underline{W_{f^{-1}} = \mathbb{R}_0^+}}$



6c

$$5 - 2\cos^2(x) - 7\sin(x) = 0$$

$$5 - 2(1 - \sin^2(x)) - 7\sin(x) = 0$$

$$5 - 2 + 2\sin^2(x) - 7\sin(x) = 0$$

$$2\sin^2(x) - 7\sin(x) + 3 = 0$$ 1P

Substitution: $a = \sin(x)$

$$2a^2 - 7a + 3 = 0$$ —

$$a_{1/2} = \frac{7 \pm \sqrt{49 - 4\cdot 2\cdot 3}}{4}$$

$$a_{1/2} = \frac{7 \pm \sqrt{25}}{4} \Rightarrow a_1 = \frac{12}{4} = 3$$ 1P

$$a_2 = 0{,}5$$

$$x_1 = \arcsin(a_1) \Rightarrow x_1 \notin \mathbb{L}$$

$$x_2 = \arcsin(0{,}5) = \frac{\pi}{6}$$ 0,5P

Periodizität

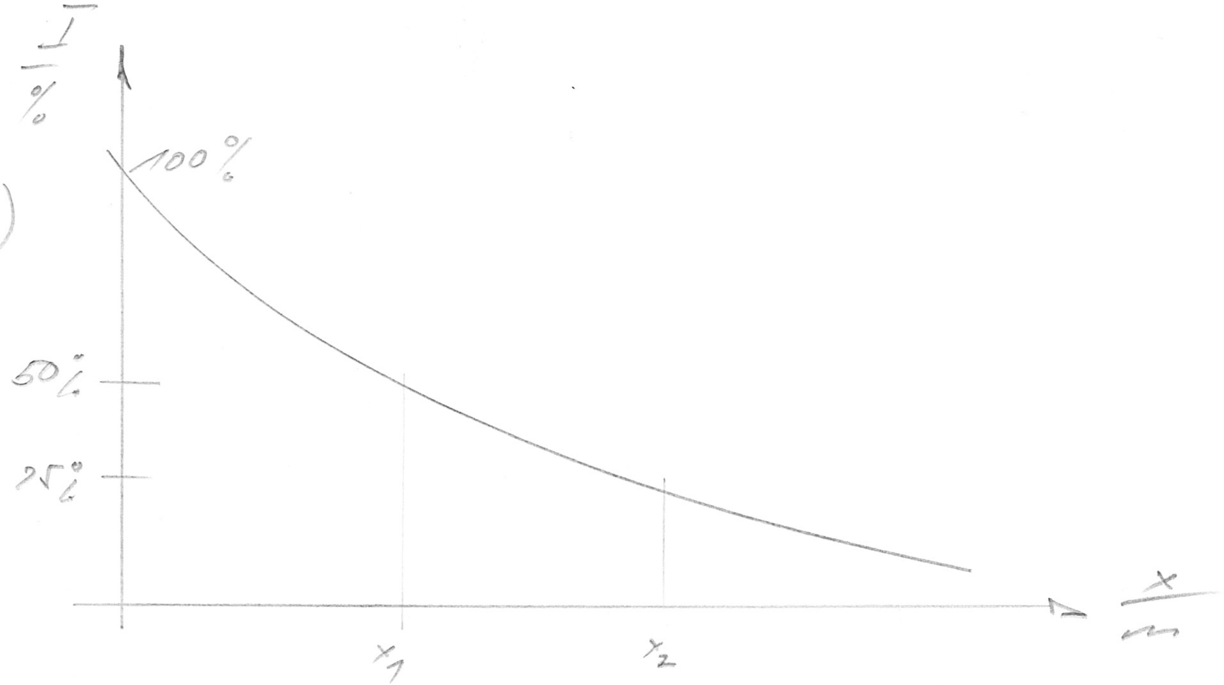
$$x_2 = \frac{\pi}{6}\ ; \quad x_3 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$ 0,5P

$$\Rightarrow \mathbb{L} = \left\{\frac{\pi}{6} ; \frac{5\pi}{6}\right\}$$

· prop. Abzüge

7

a₁)



$$\underline{\underline{I(x) = I_0 e^{-kx}}}$$

a₂) $0{,}50 = e^{-k \cdot x_1}$

$$\left.\begin{aligned} x_1 &= \frac{\ln(0{,}50)}{-k} \\ x_2 &= \frac{\ln(0{,}25)}{-k} \end{aligned}\right\} \quad \frac{x_1}{x_2} = \frac{\ln(0{,}50)}{\ln(0{,}25)}$$

$$\underline{\underline{\frac{x_1}{x_2} = \frac{1}{2}}}$$

a₃) $0{,}10 = e^{-k \cdot 1}$

$$\underline{k} = -\ln(0{,}1) = \underline{2{,}303}$$

$I = 50\% \Rightarrow \quad 0{,}50 = e^{-2{,}303x}$

$$x = \frac{\ln(0{,}5)}{-2{,}303} = 0{,}301\,m$$

$$\underline{\underline{x = 30{,}1\,cm}}$$

7b)

$$R(t) = e^{-\left(\frac{t}{T}\right)^b}$$

$$R = e^{-\left(\frac{t}{T}\right)^b}$$

$$\frac{1}{R} = e^{\left(\frac{t}{T}\right)^b}$$

$$\frac{1}{R} = e^{\frac{t^b}{T^b}} \qquad \Big| \cdot \ln()$$

$$\ln\left(\frac{1}{R}\right) = \frac{t^b}{T^b}$$

$$T^b = \frac{t^b}{\ln\left(\frac{1}{R}\right)} \qquad \Big| ()^{\frac{1}{b}}$$

$$\underline{\underline{T}} = \frac{t}{\left[\ln\left(\frac{1}{R}\right)\right]^{1/b}} = \underline{\underline{\frac{t}{\sqrt[b]{\ln\left(\frac{1}{R}\right)}}}}$$

$$\underline{\underline{T = \frac{t}{\sqrt[b]{-\ln R}}}}$$

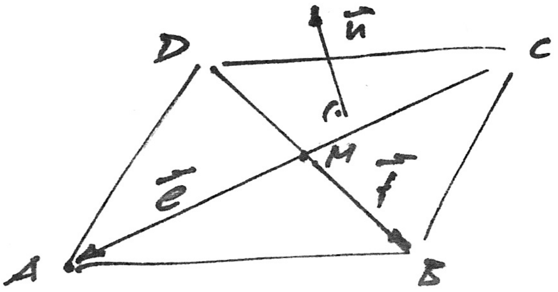
- proportionale Abzüge
- r × 0,4 r

8

$\vec{n} \times \vec{e} = k\vec{f}$ 1P

M(2|-16|8)

A(6|-12|15)



$\vec{e} = \overrightarrow{MA} = \vec{a} - \vec{m}$

$$\vec{e} = \begin{pmatrix} 6 \\ -12 \\ 15 \end{pmatrix} - \begin{pmatrix} 2 \\ -16 \\ 8 \end{pmatrix}$$

$$\underline{\vec{e} = \begin{pmatrix} 4 \\ 4 \\ 7 \end{pmatrix}}$$ 0,5P

$$k\vec{f} = \vec{n} \times \vec{e} = \begin{pmatrix} -1 \\ 8 \\ -4 \end{pmatrix} \times \begin{pmatrix} 4 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 56+16 \\ -(-7+16) \\ -4-32 \end{pmatrix} = \begin{pmatrix} 72 \\ -9 \\ -36 \end{pmatrix}$$ 0,5P

$$|k\vec{f}| = \sqrt{72^2 + (-9)^2 + (-36)^2} = 81$$ 0,5P

$$|\vec{e}| = \sqrt{16+16+49} = 9$$ 0,5P $\rightarrow \vec{f} = \frac{1}{9}\begin{pmatrix} 72 \\ -9 \\ -36 \end{pmatrix}$

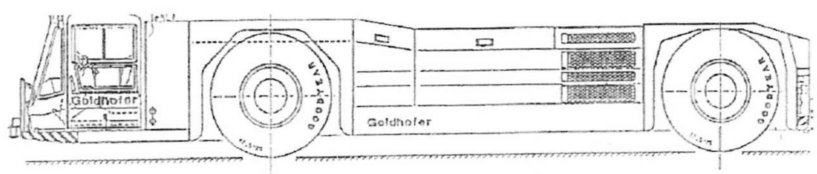
$$\vec{b} = \vec{m} + \vec{f} = \begin{pmatrix} 2+8 \\ -16-1 \\ 8-4 \end{pmatrix} \Leftarrow \underline{\vec{f} = \begin{pmatrix} 8 \\ -1 \\ -4 \end{pmatrix}}$$ 0,5P

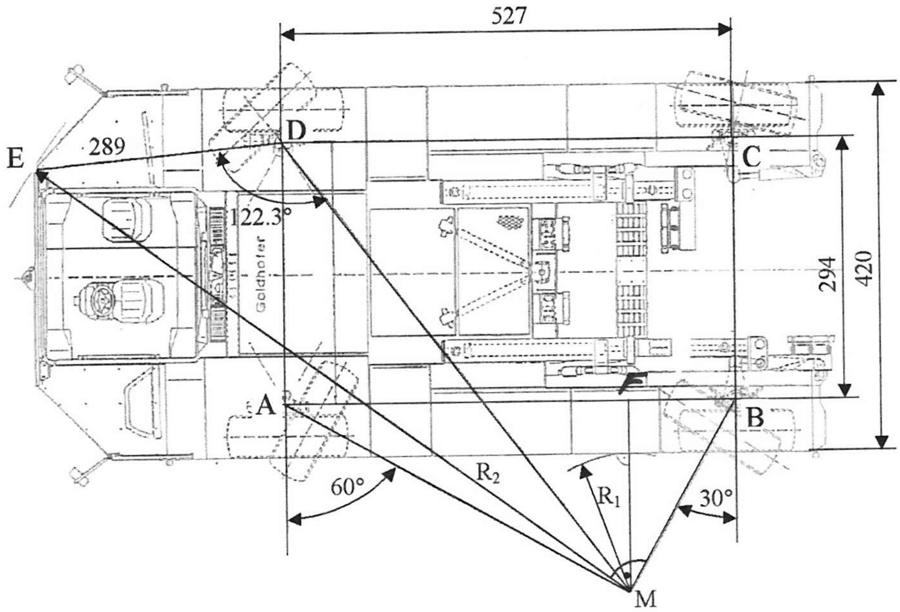
B: $\underline{\underline{\vec{b} = \begin{pmatrix} 10 \\ -17 \\ 4 \end{pmatrix}}}$ 0,5P

D: $\underline{\underline{\vec{d}}} = \vec{m} - \vec{f} = \begin{pmatrix} 2-8 \\ -16+1 \\ 8+4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -6 \\ -15 \\ 12 \end{pmatrix}}}$ 0,5P

C: $\underline{\underline{\vec{c}}} = \vec{m} - \vec{e} = \begin{pmatrix} 2 \\ -16 \\ 8 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 7 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -2 \\ -20 \\ 1 \end{pmatrix}}}$ 0,5P

9





a) $\overline{MB} = \frac{\overline{AB}}{2} = \frac{527}{2} = 263{,}5\ cm$ 0,5P

$\overline{MF} = \frac{\overline{MB}}{2}\sqrt{3} = \frac{263{,}5}{2}\sqrt{3} = 228{,}2\ cm$ 0,5P

$\underline{\underline{R_1}} = \overline{MF} - \frac{420-294}{2} = \underline{\underline{165\ cm}}$ 1P

b) $\overline{AF} = \overline{MF}\cdot\sqrt{3} = 228{,}2\cdot\sqrt{3} = 395{,}3\ cm$ 0,5P

$\overline{MD}^2 = \overline{AF}^2 + (\overline{MF} + \overline{AD})^2$

$\underline{MD} = \sqrt{395{,}3^2 + (228{,}2 + 294)^2} = \underline{654{,}9\ cm}$ 1P

$R_2^2 = \overline{DE}^2 + \overline{MD}^2 - 2\,\overline{DE}\,\overline{MD}\cos 122{,}3°$

$R_2^2 = \sqrt{289^2 + 654{,}9^2 - 2\cdot 289\cdot 654{,}9\cos 122{,}3°}$

$\underline{\underline{R_2 = 845\ cm}}$ 1,5P

· prop. [illegible]