BMP 2005
Mathematik 2

1) a)

$$\frac{1 - \frac{x^2-1}{x^2+1}}{\frac{x^2+1}{x^2-1} - 1} \cdot \left(\frac{1}{x+1} - \frac{1}{x-1}\right)$$

$$= \frac{\frac{(x^2+1)-(x^2-1)}{(x^2+1)}}{\frac{(x^2+1)-(x^2-1)}{(x^2-1)}} \cdot \left(\frac{(x-1)-(x+1)}{(x+1)(x-1)}\right)$$

$$= \frac{\frac{2}{(x^2+1)}}{\frac{2}{(x^2-1)}} \cdot \left(\frac{-2}{(x+1)(x-1)}\right)$$

$$= \frac{\cancel{2} \cdot \cancel{(x^2-1)} \cdot (-2)}{(x^2+1)\,\cancel{2}\,\cancel{(x+1)(x-1)}} = \frac{-2}{x^2+1}$$

b)

$$xy \cdot \sqrt[n]{x^{2n}+y^{2n}} \cdot \sqrt[n]{\left(\frac{x}{y}\right)^n - \left(\frac{y}{x}\right)^n}$$

$$= xy \cdot \sqrt[n]{(x^{2n}+y^{2n}) \cdot \left(\left(\frac{x}{y}\right)^n - \left(\frac{y}{x}\right)^n\right)}$$

$$= xy \cdot \sqrt[n]{(x^{2n}+y^{2n}) \cdot \left(\frac{x^n}{y^n} - \frac{y^n}{x^n}\right)}$$

$$= xy \cdot \sqrt[n]{(x^{2n}+y^{2n}) \cdot \frac{(x^{2n}-y^{2n})}{x^n y^n}}$$

$$= xy \cdot \sqrt[n]{\frac{(x^{2n}+y^{2n})(x^{2n}-y^{2n})}{x^n y^n}}$$

$$= xy \cdot \sqrt[n]{\frac{x^{4n}-y^{4n}}{x^n y^n}} = \frac{\cancel{xy}\cdot\sqrt[n]{x^{4n}-y^{4n}}}{\cancel{xy}}$$

$$= \underline{\underline{\sqrt[n]{x^{4n}-y^{4n}}}}$$

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2) $$x^2\left(\frac{2-\frac{1}{x}}{1+\frac{1}{x}}\right)\left(\frac{1}{x+1}+\frac{1}{2x-3}\right)\left(\frac{1}{x^2-4}\right)=0$$

$$\Rightarrow x \neq 0;\ x \neq -1;\ x \neq \frac{3}{2};\ x \neq \pm 2,\ x \neq 1$$

$$\underline{\underline{\mathbb{D} = \left\{ x \in \mathbb{R} \mid x \neq 0;\ x \neq -1;\ x \neq \frac{3}{2},\ x \neq 2,\ x \neq -2 \right\}}}$$

$$\frac{2-\frac{1}{x}}{1+\frac{1}{x}} = 0 \Rightarrow 2-\frac{1}{x} = 0$$

$$2x-1 = 0$$

$$x = \frac{1}{2}$$

$$\frac{1}{x+1} + \frac{1}{2x-3} = 0$$

$$\frac{1}{x+1} = -\frac{1}{2x-3}$$

$$2x-3 = -x-1$$

$$3x = 2$$

$$x = \frac{2}{3}$$

$$\underline{\underline{\mathbb{L} = \left\{ \frac{1}{2}\ ;\ \frac{2}{3} \right\}}}$$

3) $$\frac{24}{2(x^2-0{,}25)} + \frac{\sqrt{2x^2-0{,}5}-7}{\sqrt{2x^2-0{,}5}} = 7 - \frac{7}{\sqrt{2x^2-0{,}5}}$$

Setze $y = 2x^2 - 0{,}5 \quad , \quad y \neq 0$

$$\frac{24}{y} + \frac{\sqrt{y}-7}{\sqrt{y}} = 7 - \frac{7}{\sqrt{y}}$$

$$24 + \sqrt{y}(\sqrt{y}-7) = 7y - 7\sqrt{y}$$

$$24 + y - 7\sqrt{y} = 7y - 7\sqrt{y}$$

$$24 = 6y$$

$$4 = y$$

$$y = 2x^2 - 0{,}5 = 4$$

$$2x^2 = 4{,}5$$

$$x^2 = 2{,}25$$

$$x_{1/2} = \pm 1{,}5$$

$$\underline{\underline{\mathbb{L} = \{1{,}5 \,;\, -1{,}5\}}}$$

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4) $$\left| \begin{array}{c} s^2 - st - t^2 = 19 \\ s - t = 7 \end{array} \right| \Rightarrow s = 7 + t$$

$$\Rightarrow (7+t)^2 - (7+t)\cdot t - t^2 = 19$$

$$49 + 14t + t^2 - 7t - t^2 - t^2 = 19$$

$$49 + 7t - t^2 = 19$$

$$-t^2 + 7t + 30 = 0$$

$$A_{1/2} = \frac{-7 \pm \sqrt{\overset{13}{49+120}}}{-2} = \begin{cases} A_1 = -3 \\ A_2 = 10 \end{cases}$$

$s_1 = 7 + A_1 = 4$

$s_2 = 7 + A_2 = 17$

$$\underline{\underline{\mathbb{L} = \{ (4,-3) ; (17,10) \}}}$$

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5) a)

$$\log \sqrt{u} = \frac{1}{2} + \frac{1}{\log u}$$

$$\log u^{\frac{1}{2}} = \frac{1}{2} + \frac{1}{\log u}$$

$$\frac{1}{2} \log u = \frac{1}{2} + \frac{1}{\log u} \quad \Big| \cdot 2 \log u$$

$$(\log u)^2 = \log u + 2$$

$$(\log u)^2 - \log u - 2 = 0$$

$$x^2 - x - 2 = 0$$

$$x_{1/2} = \frac{1 \pm \sqrt{\overset{3}{1+8}}}{2} = \begin{cases} x_1 = 2 \\ x_2 = -1 \end{cases}$$

$\Rightarrow \log u = 2 \Rightarrow u = 10^2 = 100$

$\log u = -1 \Rightarrow u = 10^{-1} = \frac{1}{10}$

$$\underline{\underline{\mathbb{L} = \{ 100 ; \frac{1}{10} \}}}$$

b) $$v = v_E \cdot \frac{e^{kx} - e^{-kx}}{e^{kx} + e^{-kx}}$$

$$\frac{v}{v_E} = \frac{e^{kx} - \frac{1}{e^{kx}}}{e^{kx} + \frac{1}{e^{kx}}}$$

$$\frac{v}{v_E} = \frac{\frac{e^{2kx} - 1}{e^{kx}}}{\frac{e^{2kx} + 1}{e^{kx}}}$$

$$\frac{v}{v_E} = \frac{e^{2kx} - 1}{e^{2kx} + 1}$$

$$\frac{v}{v_E}\left(e^{2kx} + 1\right) = e^{2kx} - 1$$

$$\frac{v}{v_E} \cdot e^{2kx} + \frac{v}{v_E} = e^{2kx} - 1$$

$$\frac{v}{v_E} + 1 = e^{2kx}\left(1 - \frac{v}{v_E}\right)$$

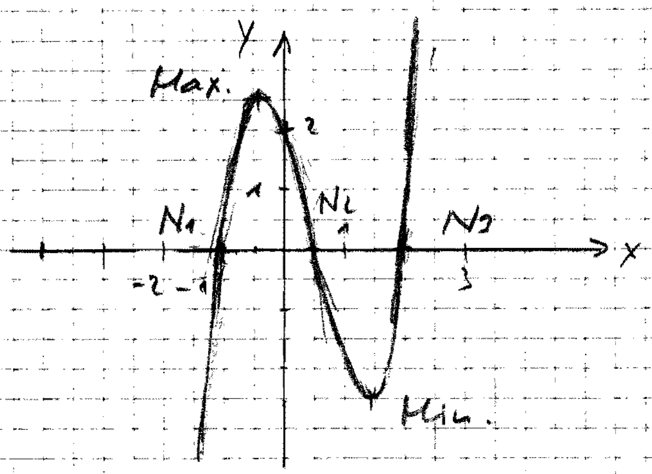
$$\frac{1 + \frac{v}{v_E}}{1 - \frac{v}{v_E}} = e^{2kx} \quad \Big| \ln$$

$$\ln\left(\frac{(v_E + v)/\cancel{v_E}}{(v_E - v)/\cancel{v_E}}\right) = 2kx$$

$$\underline{\underline{x = \frac{1}{2k} \cdot \ln\left(\frac{v_E + v}{v_E - v}\right)}}$$

6) a) geg: $f(x) = 2x^3 - 3x^2 - 3x + 2$

ges: Skizze
lokales Minimum / Maximum
Nullstelle



Minimum $(1{,}37 / -2{,}60)$
Maximum $(-0{,}37 / 2{,}60)$

Nullstellen $N_1 (-1 / 0)$
$N_2 (\frac{1}{2} / 0)$
$N_3 (2 / 0)$

b) ges: Fläche zur x-Achse geschätzt

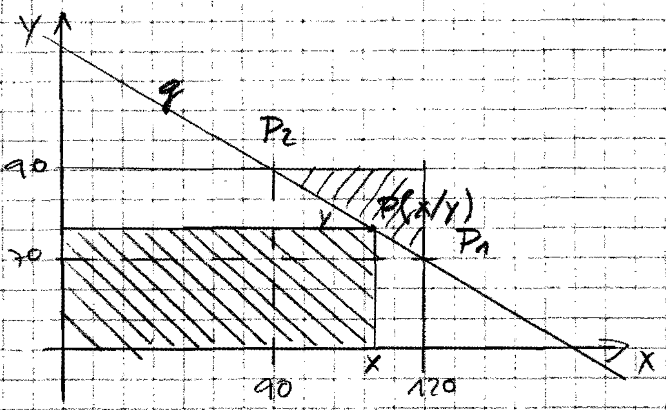
Idee: Dreieck

$A_\Delta = \frac{\Delta N \cdot y_{max}}{2} = \frac{(0{,}5 + 1) \cdot 2{,}6}{2} = 1{,}95$

$\Rightarrow A_{kurve} \approx 2$

Fläche total : ≈ 4

7)



$P \in g$ Gerade g: $y = ax + b$

$$\left| \begin{array}{l} 90 = 90a + b \\ 70 = 120a + b \end{array} \right| \Rightarrow b = 90 - 90a = 150$$

$$\text{I} - \text{II} \quad 20 = -30a$$

$$-\frac{2}{3} = a$$

$$\Rightarrow y = -\frac{2}{3}x + 150$$

Fläche $A = x \cdot y$

$$= x\left(-\frac{2}{3}x + 150\right)$$

$$= -\frac{2}{3}x^2 + 150x$$

Extremum mittels Scheitelpunkt $S\left(\frac{-b}{2a} \middle/ \frac{4ac - b^2}{2}\right)$

$\frac{-b}{2a}$: $x = \frac{-150}{2\left(-\frac{2}{3}\right)} = \underline{\underline{112{,}5 \text{ cm}}}$

$$y = -\frac{2}{3}x + 150 = \underline{\underline{75 \text{ cm}}}$$

$$\underline{\underline{A = 8437{,}5 \text{ cm}^2}}$$

8) Waldbestand

geg: $A = 0 \rightarrow$ Bestand: $20'000\,m^3$

Wachstum: $p = 2{,}5\%$

Ansatz: $y = B \cdot \left(1 + \frac{p}{100}\right)^A$

a) vor 5 Jahren: $A = -5$

$$y = B\left(1 + \frac{p}{100}\right)^{-5}$$

$$= \underline{\underline{17'677\,m^3}}$$

b)

$$y = B \cdot \left(1 + \frac{p}{100}\right)^A$$

$$\frac{y}{B} = \left(1 + \frac{p}{100}\right)^A \quad | \log$$

$$\log\left(\frac{y}{B}\right) = A \cdot \log\left(1 + \frac{p}{100}\right)$$

$$\Rightarrow A = \frac{\log\left(\frac{y}{B}\right)}{\log\left(1 + \frac{p}{100}\right)} = \underline{\underline{9{,}04 \text{ Jahren}}}$$