BMP 2005
Mathematik 1

1) geg: Parallelogramm $A(-1/-2/3)$; $B(5/4/2)$; $D(0/2/4)$.

ges: a) C

b) $A_{\square}$

a) $C = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} + \vec{AB} = \underline{\underline{\begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix}}}$

b) $A_{\square} = 2 \cdot A_{\Delta ABD}$

Sei $\vec{a} = \vec{AB} = \begin{pmatrix} 6 \\ 6 \\ -1 \end{pmatrix}$; $\vec{b} = \vec{AD} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$

$A_{\square} = 2 \cdot A_{\Delta ABD} = 2 \cdot \frac{1}{2} \sqrt{|\vec{a}|^2 \cdot |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2}$

$= \underline{\underline{21,75}}$

2) a) geg: $\vec{a} = \vec{AB} = \begin{pmatrix} 1 \\ -7 \\ -1 \end{pmatrix}$; $\vec{b} = \vec{CD} = \begin{pmatrix} -3 \\ -3 \\ 3 \end{pmatrix}$

ges: φ

$\cos\varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = 0,404$

$\Rightarrow \underline{\underline{\varphi = 66,2°}}$

b) Variante: $\vec{r}, \vec{s}$ $\quad \cos\varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = 0,2039$

$\Rightarrow \underline{\underline{\varphi = 78,23°}}$

b) geg: $\vec{u}$, $\vec{v}$, $\vec{s}$, $\vec{A}$

ges: x, y, z

$$\vec{u} = x \cdot \vec{v} + y\vec{s} + z\vec{A}$$

$$\begin{pmatrix} -11 \\ -12 \\ 1 \end{pmatrix} = x \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix} + y \begin{pmatrix} -3 \\ 5 \\ 5 \end{pmatrix} + z \begin{pmatrix} -8 \\ -6 \\ 4 \end{pmatrix}$$

$$\left| \begin{array}{l} -11 = 2x - 3y - 8z \\ -12 = 5x + 5y - 6z \\ 1 = -2x + 5y + 4z \end{array} \right|$$

$$TR \Rightarrow \quad \begin{array}{l} x = 1 \\ y = -1 \\ z = 2 \end{array}$$

$$\Rightarrow \underline{\underline{\vec{u} = \vec{v} - \vec{s} + 2\vec{A}}}$$

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(3) geg: g und h

ges: a) Schnittpunkt S

b) $\alpha \leq 90°$

a)
$$\left| \begin{array}{l} -3 + 3A = -16 + 4k \\ 5 - 5A = -6 + 1{,}5k \end{array} \right|$$

$$\left| \begin{array}{l} -4k + 3A = -13 \\ -1{,}5 - 5A = -11 \end{array} \right|$$

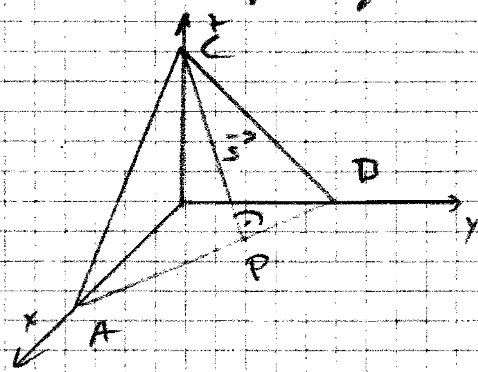
$$TR \Rightarrow k = 4, \quad A = 1$$

$$\underline{\underline{S = \begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix}}}$$

b) $\cos\varphi = \dfrac{\vec{u_1}\cdot\vec{u_2}}{|\vec{u_1}|\cdot|\vec{u_2}|} = 0$

$\Rightarrow \underline{\underline{\varphi = 90°}}$

3) geg: A, D, C Kugel von C auf xy-Ebene
ges: a) P
b) $|\vec{s}|$



a) $P \in$ Gerade durch A, D

$\Rightarrow \vec{r} = \begin{pmatrix} 8 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -8 \\ 6 \\ 0 \end{pmatrix}$

$\Rightarrow P = \begin{pmatrix} 8-8\lambda \\ 6\lambda \\ 0 \end{pmatrix}$

$\varphi = 90° \Rightarrow \vec{AB}\cdot\vec{PC} = 0$

$\begin{pmatrix} -8 \\ 6 \\ 0 \end{pmatrix}\begin{pmatrix} -8+8\lambda \\ -6\lambda \\ 0 \end{pmatrix} = 0$

$\Rightarrow -8(-8+8\lambda) - 36\lambda = 0$

$64 - 64\lambda - 36\lambda = 0$

$\lambda = 0{,}64$

$\Rightarrow \underline{\underline{P\begin{pmatrix} 2{,}88 \\ 3{,}84 \\ 0 \end{pmatrix}}}$

b) $\underline{\underline{|\vec{s}| = |\vec{PC}| = 6{,}93}}$

4) a) $$1-\sin^2(x) = 10\cdot\cos(x)-9$$
$$\cos^2(x) = 10\cos(x)-9$$

$$y=\cos(x) \Rightarrow y^2-10y+9=0$$

$$y_{1/2} = \frac{10\pm\sqrt{100-36}}{2} \quad \begin{cases} y_1 = 9 \\ y_2 = 1 \end{cases}$$

$y_1 = 9 = \cos(x) \Rightarrow$ nicht def.

$y_2 = 1 = \cos(x) \Rightarrow x = 0$

$$\Rightarrow \mathbb{L} = \{x \mid x = n\cdot 2\pi,\ n \in \mathbb{N}\}$$

b) $$\sin(\varphi) + \sin\left(\varphi+\frac{2\pi}{3}\right) + \sin\left(\varphi+\frac{4\pi}{3}\right)$$

$$= \sin(\varphi) + \sin(\varphi)\cdot\cos\left(\frac{2\pi}{3}\right) + \cos(\varphi)\sin\left(\frac{2\pi}{3}\right)$$
$$+ \sin(\varphi)\cos\left(\frac{4\pi}{3}\right) + \cos(\varphi)\sin\left(\frac{4\pi}{3}\right)$$

$$= \sin(\varphi) - \frac{1}{2}\sin(\varphi) + \frac{\sqrt{3}}{2}\cdot\cos(\varphi) - \frac{1}{2}\sin(\varphi) - \frac{\sqrt{3}}{2}\cos\varphi$$

$$= \underline{\underline{0}}$$

5) geg: Viereck ABCD

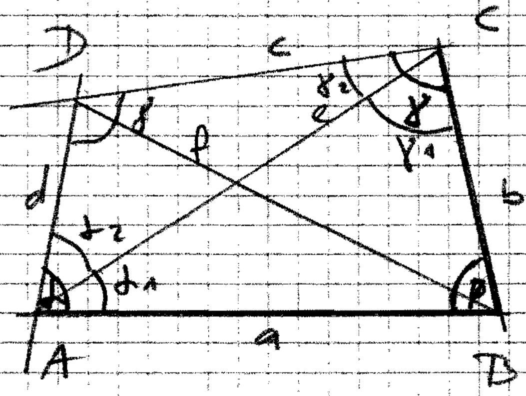
$a = 6{,}3\,cm$

$b = 5{,}4\,cm$

$\alpha = 66°$

$\beta = 78°$

$\gamma = 112°$



ges: e, c, d, f

$\delta = 104°$

$e = \sqrt{a^2 + b^2 - 2ab\cos\beta} = \underline{\underline{7,4\,cm}}$

$\frac{\sin\alpha_1}{b} = \frac{\sin\beta}{e} \Rightarrow \alpha_1 = 45,57°$

$\gamma_2 = 56,42°$

$\alpha_2 = \alpha - \alpha_1 = 20,43°$

$\gamma_2 = \gamma - \gamma_1 = 55,57°$

$c = \frac{e \cdot \sin\alpha_2}{\sin\delta} = \underline{\underline{7,66\,cm}}$

$d = \frac{e \cdot \sin\gamma_2}{\sin\delta} = \underline{\underline{6,29\,cm}}$

$f = \sqrt{b^2 + c^2 - 2bc\cos\gamma} = \underline{\underline{6,86\,cm}}$

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6)
gg: A, B, C und P, Q

ges: a) Parabel

b) S_1, S_2

a) gl. Parabel

$$\left|\begin{array}{l} 1 = 9a - 3b + c \\ 5 = a + b + c \\ 4 = 9a + 3b + c \end{array}\right|$$

TR: $a = -\frac{1}{4}$; $b = \frac{1}{2}$; $c = 4,75$

$$\underline{\underline{y = -\frac{1}{4}x^2 + \frac{1}{2}x + 4,75}}$$

b) g_1 durch P, Q

$a_1 = \frac{\Delta y}{\Delta x} = 2$

$g_2 \perp$ auf $g_1 \Rightarrow a_2 = -\frac{1}{2}$

g_2 geht durch $M(3/2)$

$\Rightarrow y_2 = -\frac{1}{2}x + b$

$2 = -\frac{1}{2} \cdot 3 + b \Rightarrow b = \frac{7}{2}$

$\Rightarrow y = -\frac{1}{2}x + \frac{7}{2}$ schneidet Parabel in S_1, S_2

$$\left| \begin{array}{l} y = -\frac{1}{4}x^2 + \frac{1}{2}x + \frac{19}{4} \\ y = -\frac{1}{2}x + \frac{7}{2} \end{array} \right|$$

TR: $x_1 = -1$ $\Rightarrow \underline{\underline{S_1(-1/4),\ S_2(5/1)}}$

$x_2 = 5$

7) geg: $U = 10$cm, Skizze

ges: a, b, A, so dass A maximal wird

$U = 10 = 6a + 2b + 2\sqrt{2}a \Rightarrow b = 5 - 3a - \sqrt{2}a$

$A = 3ab + a^2$

$= a^2 + 3a(5 - 3a - \sqrt{2}a)$

$= a^2 + 15a - 9a^2 - 3\sqrt{2}a^2$

$= -8a^2 + 15a - 3\sqrt{2}a^2 \rightarrow$ maximal

TR: $\underline{\underline{a = 0{,}613\text{cm}}}$

$\underline{\underline{b = 2{,}296\text{cm}}}$

$\underline{\underline{A = 4{,}595\text{cm}^2}}$