Aufgabe 1

x: Geschwindigkeit von Peter (km/h)

$s = v \cdot t \quad / \quad t = \frac{s}{v}$

$$\frac{7{,}175}{x} \leq \frac{8{,}2}{1{,}2x} + \frac{1}{60} \qquad 1\tfrac{1}{2}$$

$$7{,}175 \cdot 1{,}2 \cdot 60 \leq 8{,}2 \cdot 60 + 1{,}2x \qquad \tfrac{1}{2}$$

$$516{,}6 \leq 492 + 1{,}2x$$

$$24{,}6 \leq 1{,}2x$$

$$20{,}5 \leq x \qquad 1$$

Aufgabe 2

$$\sqrt{9x-2} - \sqrt{x+1} = \sqrt{16x+1}$$

$$9x - 2 - 2\cdot\sqrt{9x-2}\cdot\sqrt{x+1} + x + 1 = 16x + 1 \qquad \frac{1}{2}$$

$$10x - 1 - 2\cdot\sqrt{(9x-2)(x+1)} = 16x + 1$$

$$-2\sqrt{(9x-2)(x+1)} = 6x + 2 \qquad \frac{1}{2}$$

$$-\sqrt{(9x-2)(x+1)} = 3x + 1$$

$$(9x-2)(x+1) = 9x^2 + 6x + 1 \qquad \frac{1}{2}$$

$$9x^2 + 7x - 2 = 9x^2 + 6x + 1$$

$$x = 3 \qquad \frac{1}{2}$$

Kontrolle:

$$\sqrt{27-2} - \sqrt{3+1} = \sqrt{48+1} \qquad \frac{1}{2}$$

$$\sqrt{25} - \sqrt{4} = \sqrt{49}$$

$$5 - 2 = 7$$

$$3 = 7 \quad \text{f}$$

$$\longrightarrow \underline{\underline{L = \{\,\}}} \qquad \frac{1}{2}$$

Aufgabe 3

	Arbeitszeit alleine (Tage)	Arbeit pro Tag
Arbeiter	x	$\frac{1}{x}$
Lehrling	y	$\frac{1}{y}$

$1\frac{1}{2}$

① $1 = 12 \cdot 4 \cdot \frac{1}{x} + 12 \cdot 3 \cdot \frac{1}{y}$

② $\frac{4}{5} = 12 \cdot 2 \cdot \frac{1}{x} + 12 \cdot 4 \cdot \frac{1}{y}$

$$1 = \frac{48}{x} + \frac{36}{y}$$

$$\frac{4}{5} = \frac{24}{x} + \frac{48}{y}$$

$$-1 = -\frac{48}{x} - \frac{36}{y}$$

$$\frac{8}{5} = \frac{48}{x} + \frac{96}{y}$$

$$\frac{3}{5} = \frac{60}{y}$$

$$y = \frac{60 \cdot 5}{3}$$

$$\underline{\underline{y = 100}}$$

1

$$1 = \frac{48}{x} + \frac{36}{100}$$

$$\frac{64}{100} = \frac{48}{x}$$

$$x = \frac{48 \cdot 100}{64}$$

$$\underline{\underline{x = 75}}$$

$\frac{1}{2}$

Aufgabe 4

$$\frac{31}{5^{k-4}} = 7^{2k}$$

$$31 \cdot 5^{4-k} = 7^{2k}$$

$$\lg(31 \cdot 5^{4-k}) = \lg 7^{2k} \qquad \frac{1}{2}$$

$$\lg 31 + \lg 5^{4-k} = \lg 7^{2k} \qquad \frac{1}{2}$$

$$\lg 31 + (4-k) \cdot \lg 5 = 2k \cdot \lg 7 \qquad \frac{1}{2}$$

$$\lg 31 + 4\lg 5 - k \cdot \lg 5 = 2k \cdot \lg 7$$

$$\lg 31 + 4 \cdot \lg 5 = 2k \cdot \lg 7 + k \cdot \lg 5 \qquad \frac{1}{2}$$

$$\lg 31 + 4 \cdot \lg 5 = k \cdot (2 \lg 7 + \lg 5)$$

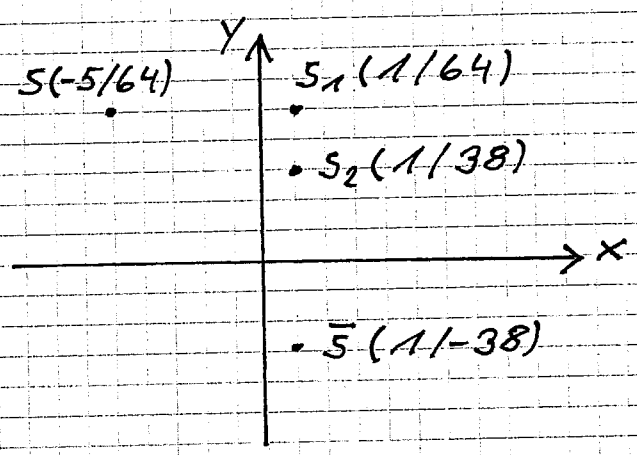
$$k = \frac{\lg 31 + 4 \cdot \lg 5}{2 \cdot \lg 7 + \lg 5} \qquad \frac{1}{2}$$

$$k = 1{,}7945$$

$$\underline{\underline{L = \{1{,}7945\}}} \qquad \frac{1}{2}$$

Aufgabe 5

a)



$$\bar{F}(x) = \frac{1}{3}(x-1)^2 - 38$$ 1

b) $$-\frac{1}{3}(x+5)^2 + 64 = \frac{1}{3}(x-1)^2 - 38$$ $\frac{1}{2}$

$$-(x+5)^2 + 192 = (x-1)^2 - 114$$

$$-x^2 - 10x - 25 + 192 = x^2 - 2x + 1 - 114$$

$$-x^2 - 10x + 167 = x^2 - 2x - 113$$

$$0 = 2x^2 + 8x - 280$$

$$0 = x^2 + 4x - 140$$ $\frac{1}{2}$

$$x_{1;2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot (-140)}}{2 \cdot 1}$$

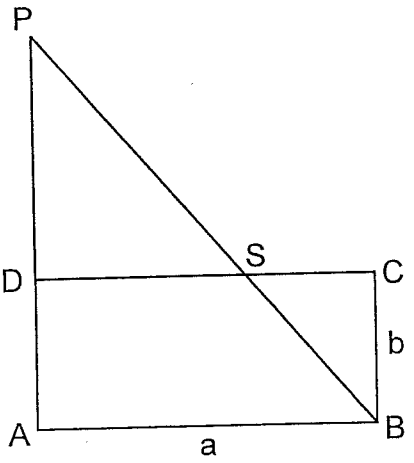
$$= \frac{-4 \pm 24}{2}$$

$$x_1 = \frac{20}{2} = 10 \longrightarrow y_1 = -\frac{1}{3}(10+5)^2 + 64 = -11$$ $\frac{1}{2}$

$$x_2 = \frac{-28}{2} = -14 \longrightarrow y_2 = -\frac{1}{3}(-14+5)^2 + 64 = 37$$

$P_1(10/-11)$ $\quad$ $P_2(-14/37)$ $\frac{1}{2}$

Aufgabe 6



a) $A_{ABSD} = \frac{10}{11} ab = b \cdot \frac{1}{2}(a + \overline{DS})$ $\frac{1}{2}$

$20a = 11(a + \overline{DS})$

$20a = 11a + 11\overline{DS}$ $\frac{1}{2}$

$9a = 11\overline{DS}$

$\overline{DS} = \frac{9}{11} a$ $\frac{1}{2}$

b) $\overline{PD} : \overline{PA} = \frac{9}{11} a : a$ $\frac{1}{2}$

$\overline{PD} : (\overline{PD} + b) = 9 : 11$

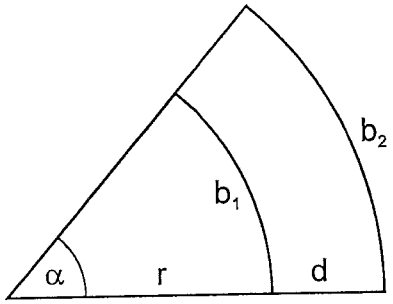
$11\overline{PD} = 9(\overline{PD} + b)$ $\frac{1}{2}$

$11\overline{PD} = 9\overline{PD} + 9b$

$2\overline{PD} = 9b$

$\overline{PD} = \frac{9}{2} b$ $\frac{1}{2}$

Aufgabe 7



① $\frac{2r\pi\alpha}{360°} = 15 \longrightarrow \alpha = \frac{15 \cdot 360°}{2r\pi}$

② $\frac{2(r+4)\pi \cdot \alpha}{360°} = 20 \longrightarrow \alpha = \frac{20 \cdot 360°}{2 \cdot (r+4)\pi}$

$$\frac{15 \cdot 360°}{2r\pi} = \frac{20 \cdot 360°}{2(r+4) \cdot \pi}$$

$$15 \cdot (r+4) = 20r$$

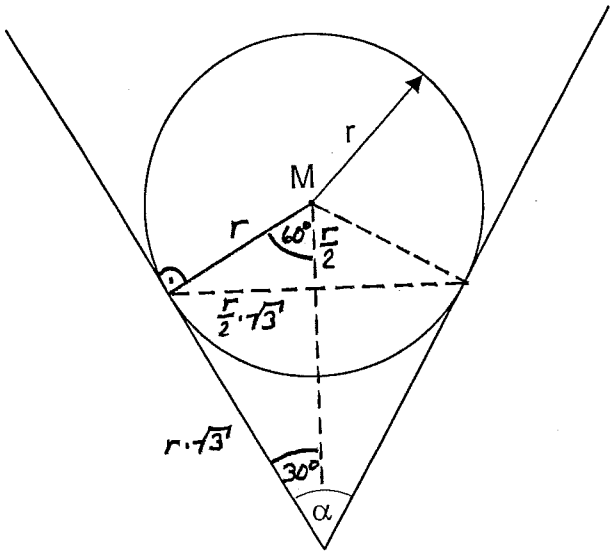
$$15r + 60 = 20r$$

$$60 = 5r$$

$$\underline{\underline{r = 12\ cm}}$$ 2

$$\alpha = \frac{15 \cdot 360°}{2 \cdot 12 \cdot \pi} = \underline{\underline{71{,}62°}}$$ 1

Aufgabe 8



1/2

$$V_{Kegel} = \frac{\left(\frac{r}{2}\sqrt{3}\right)^2 \cdot \pi \cdot \frac{3}{2}r}{3} = \frac{\frac{r^2}{4} \cdot 3 \cdot \pi \cdot \frac{3}{2}r}{3}$$

$$= \frac{r^3 \pi \cdot 3}{8}$$

1

$$V_{Kugelsegment} = \frac{\pi \cdot \left(\frac{r}{2}\right)^2}{3} \cdot \left(3r - \frac{r}{2}\right)$$

$$= \frac{\pi \cdot \frac{r^2}{4}}{3} \cdot \frac{6r - r}{2} = \frac{\pi \cdot r^2 \cdot 5r}{3 \cdot 4 \cdot 2}$$

$$= \frac{r^3 \pi \cdot 5}{24}$$

1

$$V = \frac{r^3 \pi \cdot 3}{8} - \frac{r^3 \pi \cdot 5}{24} = \frac{r^3 \cdot \pi \cdot 4}{24}$$

$$= \frac{r^3 \pi}{6}$$

1/2

Aufgabe 9

a) $A = \frac{a \cdot c \cdot \sin\beta}{2}$

$$c = \frac{2A}{a \cdot \sin\beta} = \underline{\underline{15{,}45\,cm}} \qquad 1$$

$$b = \sqrt{a^2 + c^2 - 2ac\cos\beta} = \underline{\underline{25{,}38\,cm}} \qquad \frac{1}{2}$$

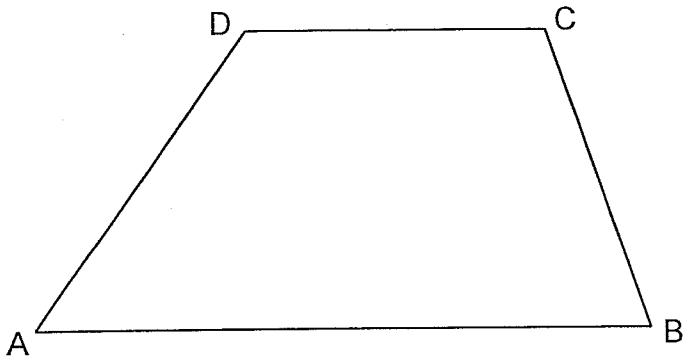
b) $\frac{\sin\alpha}{\sin\beta} = \frac{a}{b}$

$$\sin\alpha = \frac{a}{b} \cdot \sin\beta = 0{,}89$$

$$\underline{\underline{\alpha = 63{,}17°}} \qquad 1$$

$$\gamma = 180° - \alpha - \beta = \underline{\underline{36{,}83°}} \qquad \frac{1}{2}$$

Aufgabe 10



$$\overrightarrow{AB} = \begin{pmatrix} 5-17 \\ -2-(-18) \\ 3-(-1) \end{pmatrix} = \begin{pmatrix} -12 \\ 16 \\ 4 \end{pmatrix}$$

$\frac{1}{2}$

$$C: \vec{r} = \begin{pmatrix} 13 \\ -12 \\ 2 \end{pmatrix} + t \cdot \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix}$$

$$\overrightarrow{BC} = \begin{pmatrix} 13-3t-5 \\ -12+4t-(-2) \\ 2+t-3 \end{pmatrix} = \begin{pmatrix} 8-3t \\ -10+4t \\ -1+t \end{pmatrix} \qquad |\overrightarrow{BC}| = 3$$

$\frac{1}{2}$ $\frac{1}{2}$

$$9 = (8-3t)^2 + (-10+4t)^2 + (-1+t)^2$$

$\frac{1}{2}$

$$9 = 64 - 48t + 9t^2 + 100 - 80t + 16t^2 + 1 - 2t + t^2$$

$$0 = 26t^2 - 130t + 156$$

$\frac{1}{2}$

$$0 = t^2 - 5t + 6$$

$$t_{1/2} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1} = \frac{5 \pm 1}{2}$$

$t_1 = 3$

$$C_1: \vec{r} = \begin{pmatrix} 13 \\ -12 \\ 2 \end{pmatrix} + 3 \cdot \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix}$$

$\frac{1}{2}$

$C_1 = (4/0/5)$

$t_2 = 2$

$$C_2: \vec{r} = \begin{pmatrix} 13 \\ -12 \\ 2 \end{pmatrix} + 2 \cdot \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ -4 \\ 4 \end{pmatrix}$$

$\frac{1}{2}$

$C_2 = (7/-4/4)$