Allgemeine Gewerbeschule Basel
BERUFSMATURITÄTSSCHULE

Maturprüfung 2006
Mathematik

Lösungsschlüssel

Aufgabe 1 **4 P.**

cos-Satz: $a = a_1 + a_2$; $b = b_1 + b_2$; $c = c_1 + c_2$

$$\cos\beta = \frac{b^2 - a^2 - c^2}{-2ac} = 0{,}845608 \Rightarrow \underline{\beta = 32{,}2629°}$$

↳ 1/2 P ↳ 1/4 P ↳ 1/4 P

$$x^2 = a_1^2 + c_2^2 - 2a_1c_2\cos\beta = 7'004{,}315$$

↳ 1/2 P ↳ 1/4 P

$\underline{\underline{x = 83{,}6918\,m}}$ → 1/4 P

sin-Satz:

$$\sin\psi = \frac{c \cdot \sin\beta}{b} = 0{,}817388 \Rightarrow \underline{\psi = 125{,}1758°}$$

↳ 1/2 P ↳ 1/4 P ↳ 1/4 P

cos-Satz:

$$y^2 = a_2^2 + b_1^2 - 2a_2b_1\cos\psi = 8735{,}869564$$

↳ 1/2 P ↳ 1/4 P

$\underline{\underline{y = 93{,}4659\,m}}$ → 1/4 P

Aufgabe 2 — 4 P.

a) $\dfrac{a^n(a^{2n}-1)}{a^{n-1}(a^n+1)} =$ — ½ P, ½ P — 1 P

$\dfrac{a^n(a^n-1)(a^n+1)}{a^{n-1}(a^n+1)} =$ — ½ P

$\underline{a(a^n-1)}$ — ½ P

b) $a \cdot \sqrt[k]{\dfrac{a}{a^k} \cdot \sqrt[k]{\dfrac{a}{a^k}}} = \sqrt[x]{a}$ — ½ P

$\sqrt[k]{\dfrac{a^k \cdot a}{a^k} \cdot \sqrt[k]{\dfrac{a}{a^k}}} = \sqrt[x]{a}$ — ½ P

$\sqrt[k^2]{\dfrac{a^k \cdot a}{a^k}} = \sqrt[x]{a}$ — ½ P

$\underline{x = k^2}$ — ½ P

Aufgabe 3 **4 P.**

a.) $(30°/60°/90°)$-Dreieck $\Rightarrow \underline{\underline{\ell_1 = 2a}}$ → 1/2 P

b.) $\ell_2^2 = \ell_1^2 + 1/4(\ell_1 + \ell_2)^2 = 5/4\,\ell_1^2 + 1/2\,\ell_1 \cdot \ell_2 + 1/4\,\ell_2^2$

mit $\ell_1 = 2a$ folgt → 3/4 P

$\ell_2^2 = 5a^2 + a \cdot \ell_2 + 1/4\,\ell_2^2$ → 1/2 P

$3/4\,\ell_2^2 - a \cdot \ell_2 - 5a^2 = 0$ → 1/2 P

$$\ell_2 = \frac{a \pm \sqrt{a^2 - 4 \cdot 3/4 \cdot (-5a^2)}}{1{,}5}$$

$$\ell_2 = \frac{a \pm 4a}{1{,}5} = \begin{cases} \underline{\underline{\ell_2 = 3\,1/3\,a}} \\ \ell_2 = -2a \end{cases}$$

→ 1/2 P

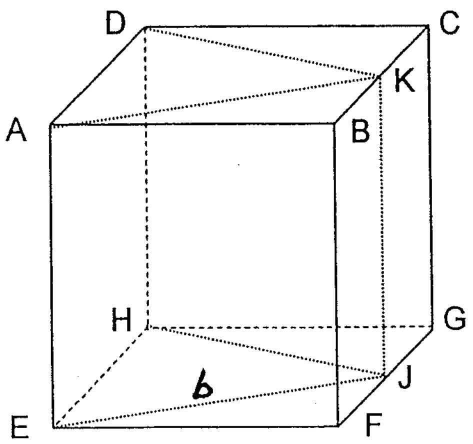
$\ell_2 = -2a$ ↳ keine Lösung! } 1/4 P

c.) $V_P = \frac{A \cdot h}{3}$ → 1/4 P

$$\underline{\underline{V_P}} = \frac{1}{3} \cdot \underbrace{\frac{8}{3}a \cdot \frac{1}{2}a}_{1/4\,P} \cdot \underbrace{a\sqrt{3}}_{1/4\,P} = \underline{\underline{\frac{4\sqrt{3}}{9}a^3}}$$

↳ 1/4 P

Aufgabe 4 **4 P.**



1,118a

$b = \sqrt{a^2 + a^2/4} = \frac{a}{2}\sqrt{5}$ *(Strecke EJ)*

½ P ½ P

$a)$ *Länge aller Kanten*: $5a + 4b = 5a + 4\frac{a}{2}\sqrt{5} = \underline{\underline{5a + 2a\sqrt{5}}} = \underline{a(5+2\sqrt{5})}$ $9{,}472a$

½ P ½ P

$b)$ *Oberfläche*: $a^2 + 2ab + 2\frac{a^2}{2} = a^2 + 2a\cdot\frac{a}{2}\sqrt{5} + a^2 = \underline{\underline{a^2(2+\sqrt{5})}} = 4{,}236a^2$

½ P ½ P

$c)$ *Volumen*: $Gfl \times h = \frac{a^2}{2}(\text{halbe Gfl des Würfels}) \times a = \underline{\underline{\frac{a^3}{2}}}$

½ P ½ P

Aufgabe 5 **4 P.**

a) Lin. Faktoren:

$(x+1)(x-3{,}5)=0$

$x^2-2{,}5x-3{,}5=0$

→ je 1/2 P

$y=x^2-2{,}5x-3{,}5$

→ 1/2 P

Satz von Vieta:

$x_1+x_2=-b \Rightarrow b=-2{,}5$

$x_1 \cdot x_2 = c \Rightarrow c=-3{,}5$

→ je 1/2 P

$y=x^2-2{,}5x-3{,}5$

→ 1/2 P

1½

b) $y=2x-3$ mit $x=-2 \Rightarrow y=-7$ → 1/4 P

$y=x^2-2{,}5x+c$ → 1/2 P

$-7=4+5+c \Rightarrow c=-16$

→ 1/4 P

!

$y=x^2-2{,}5x-16$ → 1/4 P

1¼

c) $ax^2-2{,}5x-3{,}5=2x-3$ → 1/4 P

$ax^2-4{,}5x-0{,}5=0$ → 1/4 P

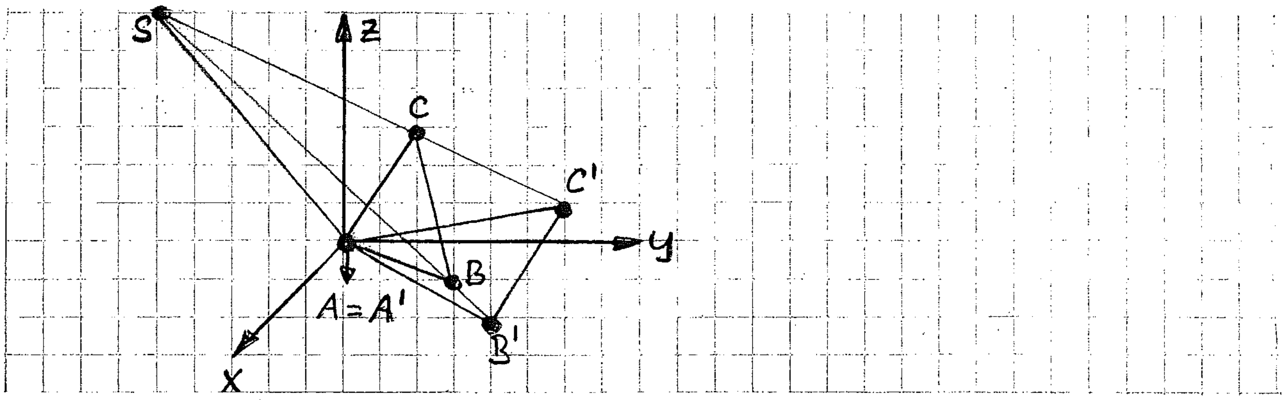
Diskriminante $=0$:

$D=b^2-4ac=0 \Rightarrow 4{,}5^2+2a=0$ → 1/2 P

$a=-10{,}125$ → 1/4 P

1¼

Aufgabe 6 — 4 P.



a) $\underline{A'=(0/0/0)}$ (logisch). → ½ P.

Gerade SB: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 5 \\ 2 \end{pmatrix} + t \begin{pmatrix} 6 \\ -6 \\ 12 \end{pmatrix}$ Spurpunkt S_{xy} → z=0 → 1/4 P

$\Rightarrow 0 = 2 + 12t$

→ $\underline{t = -1/6}$ → $\underline{x=7, \; y=6}$ → $\underline{B'=(7/6/0)}$ — 1P.

↳ 1/4 P ↳ ↳ je 1/4 P

Gerade SC: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 7 \end{pmatrix} + t \begin{pmatrix} 10 \\ -5 \\ 7 \end{pmatrix}$ Spurpunkt S_{xy} → z=0 → 1/4 P

$\Rightarrow 0 = 7 + 7t$

→ $\underline{t = -1}$ → $\underline{x=-6, \; y=9}$ → $\underline{C'=(-6/9/0)}$ — 1P

↳ 1/4 P ↳ ↳ je 1/4 P

b) $\overrightarrow{C'A'} = \begin{pmatrix} 0-(-6) \\ 0-9 \end{pmatrix} = \underline{\begin{pmatrix} 6 \\ -9 \end{pmatrix}}$, $\left|\overrightarrow{C'A'}\right| = \underline{\sqrt{117}}$ → 1/4 P

$\overrightarrow{C'B'} = \begin{pmatrix} 7-(-6) \\ 6-9 \end{pmatrix} = \underline{\begin{pmatrix} 13 \\ -3 \end{pmatrix}}$, $\left|\overrightarrow{C'B'}\right| = \underline{\sqrt{178}}$ → 1/4 P — 1P

$\cos\gamma' = \dfrac{6 \cdot 13 + (-9)(-3)}{\sqrt{117} \cdot \sqrt{178}} = \underline{0{,}727589}$ → $\underline{\gamma' = 43{,}3153°}$ → 1/4 P

↳ 1/4 P

c) $A_{A'B'C'} = ½\, ab \sin\gamma'$ → $A_{A'B'C'} = ½ \sqrt{117} \cdot \sqrt{178} \sin 43{,}315°$ → 1/4 P

→ $\underline{A_{A'B'C'} = 49{,}5}$ → 1/4 P

Aufgabe 7 — 4 P.

(1): $(a + 2.75)(h - 4.25) = a\cdot h - 46$ → ½ P
(2): $(a - 5.5)(h + 12) = a\cdot h + 49$ → ½ P

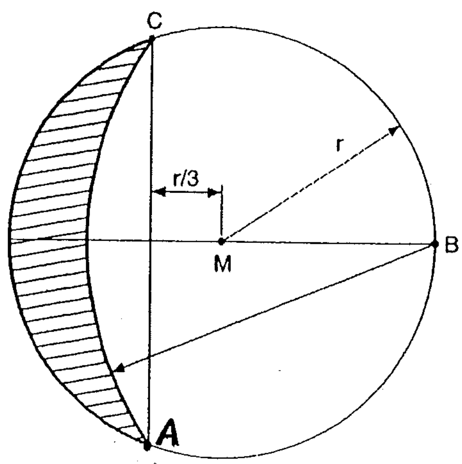
(1): $a\cdot h - 4.25a + 2.75h - 11.6875 = a\cdot h - 46$ → 1/4 P
(2): $a\cdot h + 12a - 5.5h - 66 = a\cdot h + 49$ → 1/4 P

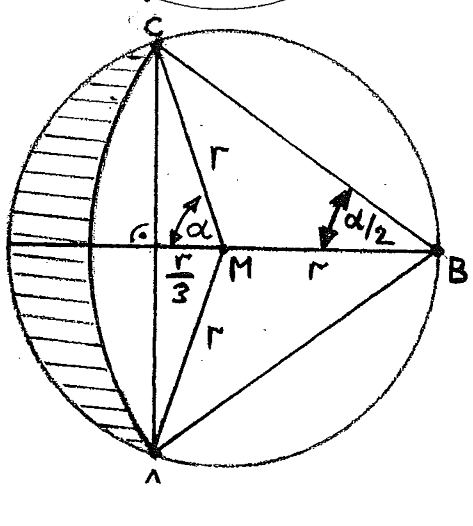
(1): $-4.25a + 2.75h = -34.3125$ → 1/4 P
(2): $12a - 5.5h = 115$ → 1/4 P

(1)·2: $-8.5a + 5.5h = -68.625$
(2): $12a - 5.5h = 115$
} → 1/2 P
(+): $3.5a = 46.375 \Rightarrow \underline{a = 13.25\text{m}}$ → 1/2 P

(2): $5.5h = 12a - 115$ → 1/2 P
(2): $5.5h = 12\cdot 13.25 - 115 = 44 \Rightarrow \underline{h = 8\text{m}}$ → 1/2 P

Aufgabe 8 — 4 P.





$\overline{AC}/2 = \sqrt{r^2 - r^2/9} = \frac{2\sqrt{2}}{3}r$ → ½ P

$\overline{AB} = \sqrt{(16/9)r^2 + (8/9)r^2}$

$= \sqrt{(24/9)r^2} = (2\sqrt{6}/3)r$ → ½ P

$\tan(\alpha/2) = \frac{2\sqrt{2}/3}{4/3} = \frac{2\sqrt{2}}{4} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \alpha = 70{,}5288°$ → ½ P

$A = \underbrace{\frac{r^2\pi 2\alpha}{360}} - \underbrace{\frac{2\sqrt{2}r^2}{9}} - \underbrace{\frac{24r^2\pi\alpha}{9\cdot 360}} + \underbrace{\frac{8\sqrt{2}r^2}{9}}$ 4x ½ P

$= \frac{6\sqrt{2}r^2}{9} - \frac{2r^2\pi\alpha}{3\cdot 360} = \frac{2}{3}(\sqrt{2} - \frac{\pi\alpha}{360})r^2$

$\underline{A = 0{,}5325r^2}$ → ½ P