1a

$x \in [0;2\pi]$

$$2\cos^2(x) + \cos(x) - 1 > 0$$

$\cos(x) = m$ 0,5P

$$2m^2 + m - 1 > 0$$

$$\Rightarrow \quad 2m^2 + m - 1 = 0$$

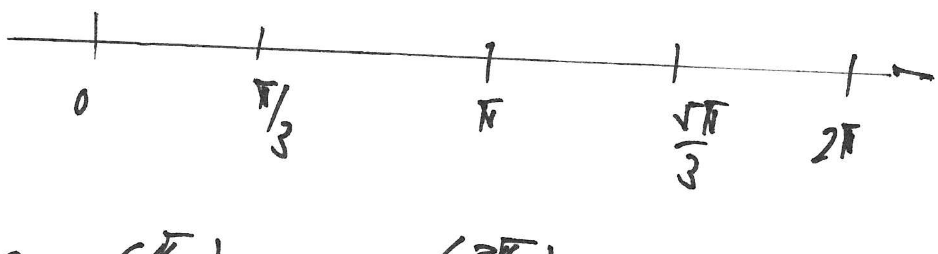
$$(2m-1)(m+1) = 0$$

$$m_1 = \frac{1}{2} \quad , \quad m_2 = -1$$ 1P

$$x_1 = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$$ 0,5P

$$x_3 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$ 0,5P

$$x_2 = \arccos(-1) = \pi$$ 0,5P



$$\cos\left(\frac{\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right) = 0 \Rightarrow -1 < 0 \,!$$ 0,5P

$$L = \left]0;\frac{\pi}{3}\right[\cup \left]\frac{5\pi}{3};2\pi\right[$$ 0,5P

16

$P(a+2 / a^2+12a-2)$

$y = x^2+6x-3$

$\Rightarrow \quad a^2+12a-2 = (a+2)^2+6(a+2)-3$

$\cancel{a^2}+12a-2 = \cancel{a^2}+4a+4+6a+12-3$

$2a = 15$

$\underline{\underline{a = 7{,}5}}$

$P[(7{,}5+2) / (7{,}5^2+12\cdot 7{,}5-2)]$

$\underline{\underline{P(9{,}5 / 144{,}25)}}$

1c

$$\left| \begin{array}{l} 9 \cdot 3^{x-2} = 9^{y-1} \\ 4^{x-3} = 4 \cdot 2^{y-3} \end{array} \right|$$

$$\Rightarrow \left| \begin{array}{l} 3^2 \cdot 3^{x-2} = 3^{2(y-1)} \\ 2^{2(x-3)} = 2^2 \cdot 2^{y-3} \end{array} \right|$$

$$\Rightarrow \left| \begin{array}{l} 3^{x} = 3^{2y-2} \\ 2^{2x-6} = 2^{y-1} \end{array} \right| \Rightarrow \begin{array}{l} x = 2y-2 \\ 2x-6 = y-1 \end{array}$$

$$- \left| \begin{array}{l} 2x = 4y-4 \\ 2x = y+5 \end{array} \right|$$

$$0 = 3y - 9$$

$$\underline{\underline{y = 3}} \quad , \quad \underline{\underline{x}} = 2 \cdot 3 - 2 = \underline{\underline{4}}$$

1d

$$\left| \begin{array}{l} x+y+z=1 \\ y-z=2 \end{array} \right| \quad \begin{array}{l} 1) \\ 2) \end{array}$$

$$\underline{\underline{z=\lambda}} \quad , \lambda \in \mathbb{R} \qquad 1P$$

2) $\underline{\underline{y}} = 2+z = \underline{\underline{2+\lambda}}$ ½P

1) $x + 2+z+z = 1$

$$x = -1-2z$$

$$\underline{\underline{x = -1-2\lambda}} \qquad ½P$$

$$\underline{\underline{\mathbb{L} = \{(-1-2\lambda\,;\,2+\lambda\,;\,\lambda)\}}}$$

2 Bedingungen für das Planungspolygon

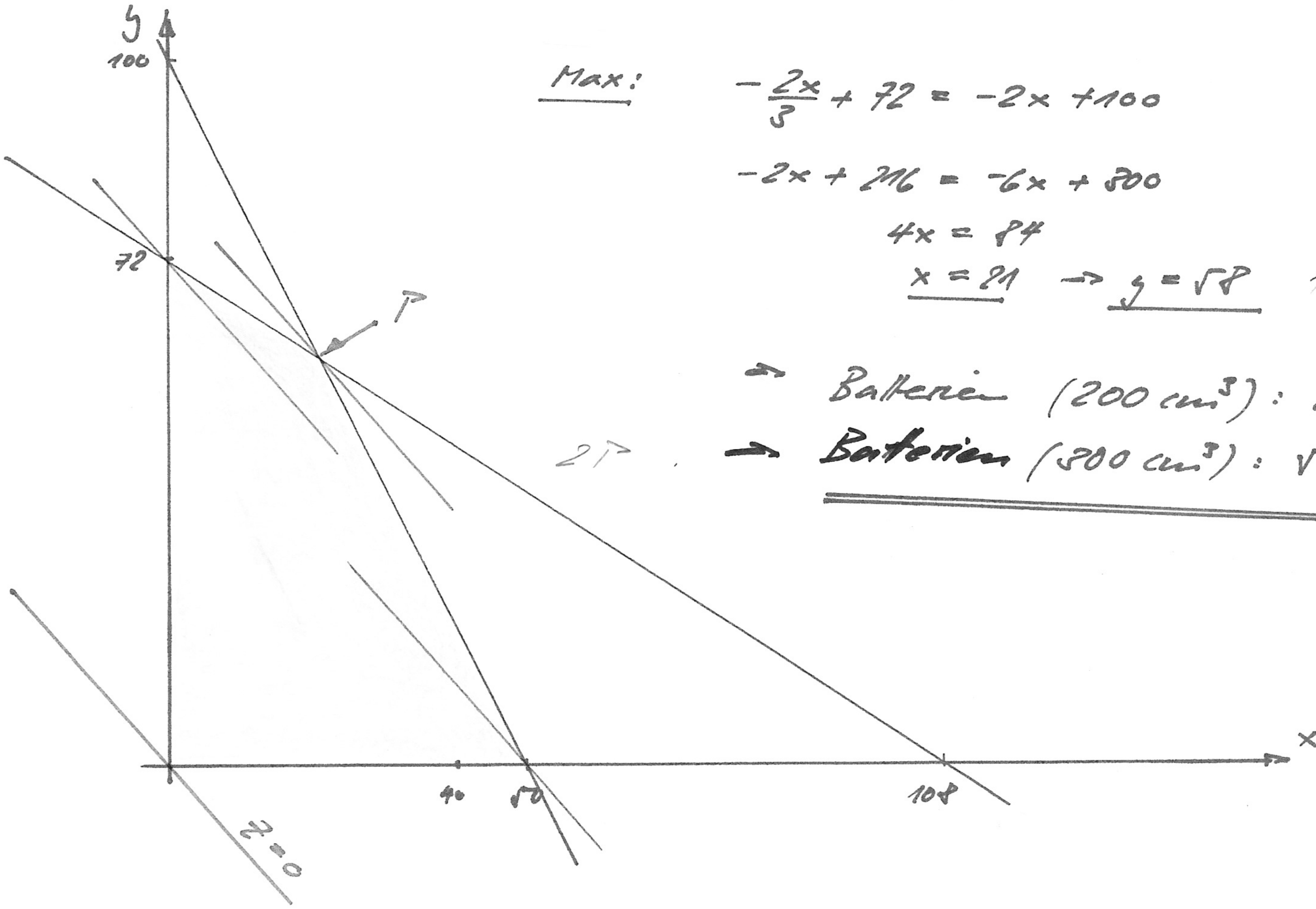
Stückzahl an Batterien der ersten Sorte : x

Stückzahl " " " zweiten " : y   0,5 P

$$\left|\begin{array}{l} x, y \geq 0 \\ 200x + 300y \leq 21'600 \\ 10x + 5y \leq 500 \end{array}\right| \quad \begin{array}{l} \\ 0{,}5\,P \\ 0{,}5\,P \end{array}$$

	x_0	y_0
$x \geq 0$		
$y \geq 0$		
$y \leq -\frac{2x}{3} + 72$ (0,5 P)	108	72
$y \leq -2x + 100$ (0,5 P)	50	10

Zielfunktion (Energiemenge) $z = 18x + 16y$ (Maximum) 0,5 P $\Rightarrow$ $y = -\frac{9x}{8} + \frac{z}{16}$ 0,5 P



Max:

$$-\frac{2x}{3} + 72 = -2x + 100$$

$$-2x + 216 = -6x + 300$$

$$4x = 84$$

$$x = 21 \rightarrow y = 58$$ 1 P

$\Rightarrow$ Batterien (200 cm³) : 21 Stk

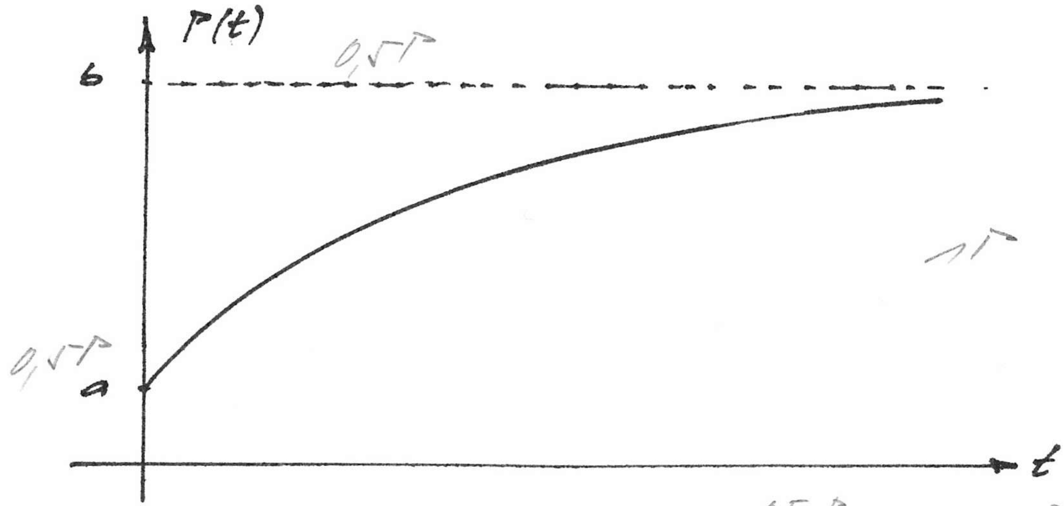
2 P $\Rightarrow$ **Batterien (300 cm³) : 58 Stk**

Energiemenge: 0,5 P

$$z = 18 \cdot 21 + 16 \cdot 58 = 1306\,h$$

<u>3a</u> $P(t) = b - (b-a)e^{-\lambda t}$

a1)



a2) $P(5) = 250$, $b = 1000$, $a = 200$ 0,5P 0,5P

$\rightarrow$ $250 = 1000 - (1000-200)e^{-5\lambda}$

$800 e^{-5\lambda} = 750$

$e^{-5\lambda} = \frac{15}{16}$ $\quad / \ln()$

$-5\lambda = \ln\left(\frac{15}{16}\right)$

$\underline{\underline{\lambda}} = \frac{-1}{5}\cdot\ln\left(\frac{15}{16}\right) = \underline{\underline{0{,}0129}}$ 2P

a3) $P = b - (b-a)e^{-\lambda t}$

$(b-a)e^{-\lambda t} = b - P$

$\left(\frac{b-a}{b-P}\right) = e^{\lambda t}$ $\quad / \ln()$

$\lambda t = \ln\left(\frac{b-a}{b-P}\right)$ 1,5P

$t = \frac{1}{\lambda}\cdot\ln\left(\frac{b-a}{b-P}\right) = \frac{1}{0{,}0129}\cdot\ln\left(\frac{1000-200}{1000-400}\right)$

$\underline{\underline{t = 22{,}3a}}$ 0,5P

3b

$$\log_a(x^2) - \log_a(x-2) = 5\log_a(3) - \log_a(27)$$

$$\log_a\left(\frac{x^2}{x-2}\right) = \log_a(3^5) - \log_a(3^3)$$

$$\log_a\left(\frac{x^2}{x-2}\right) = \log_a\left(\frac{3^5}{3^3}\right) \quad \Big/ a^{()}$$

2,5 P

$$\frac{x^2}{x-2} = 3^2$$

$$x^2 = 9(x-2)$$

$$x^2 - 9x + 18 = 0$$

$$(x-3)(x-6) = 0$$

$$\underline{\underline{x_1 = 3}}$$

$$\underline{\underline{x_2 = 6}}$$

x_1, x_2 sind nicht von der Logarithmen-basis a abhängig.

0,5 P

4a

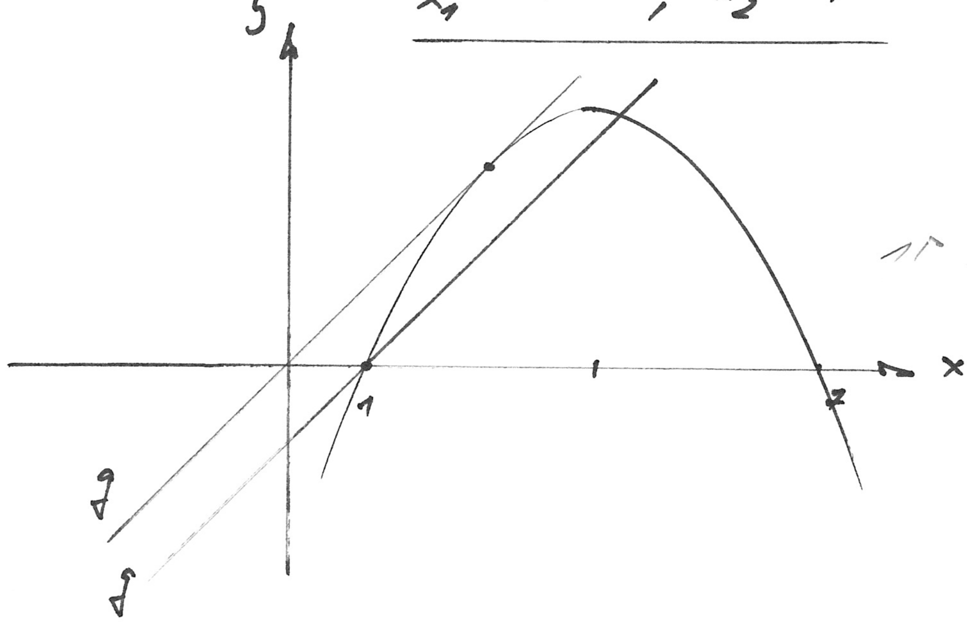
$y = p(x) = -x^2 + 8x - 7$

$g(x) = 1{,}5x + q$

Nullstellen $p(x)$:

$$-x^2 + 8x - 7 = 0$$
$$x^2 - 8x + 7 = 0$$
$$(x - 1)(x - 7) = 0$$

$x_1 = 1$, $x_2 = 7$ 1P



1P

1) $g(1) = 0 \Rightarrow 1{,}5 + q = 0$

$q = -1{,}5$ 0,5P

2) $-x^2 + 8x - 7 = 1{,}5x + q$

$x^2 - 6{,}5x + 7 + q = 0$

$\Delta = 0 \Rightarrow b^2 - 4ac = 0$

$(-6{,}5)^2 - 4 \cdot 1 \cdot (7 + q) = 0$

$25 - 28 - 4q = 0 \Rightarrow q = 0{,}75$

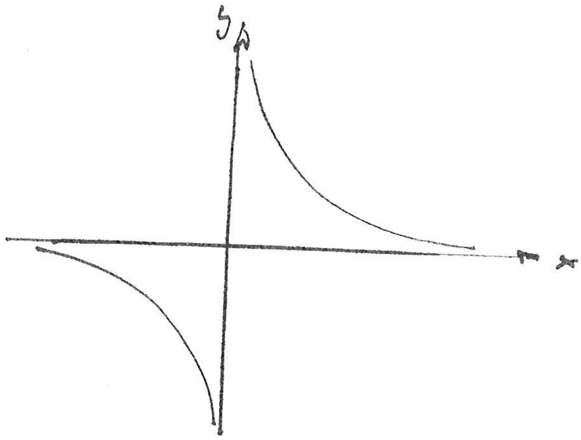
1P

0,5P

$q \in \left]-1{,}5 ; 0{,}75\right[$

4b

$$y = f(x) = \frac{a}{x}$$



1) VAS : $x = 2$

$\Rightarrow \quad y = f(x) = \dfrac{a}{x-2}$ 0,5 P

2) HAS : $y = 1$

$\Rightarrow \quad y = f(x) = \dfrac{a}{x-2} + 1$ 0,5 P

3) $f(3) = 0$

$\Rightarrow \quad 0 = \dfrac{a}{3-2} + 1 \Rightarrow \underline{a = -1}$ 0,5 P

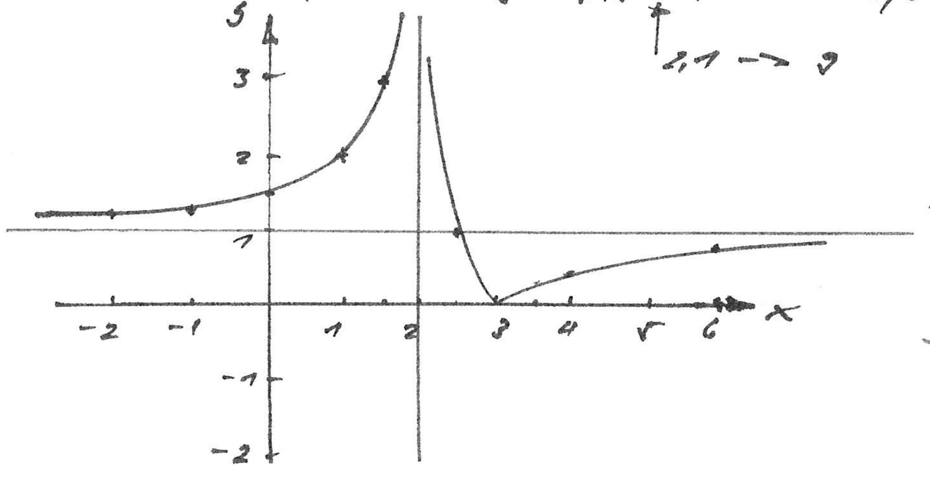
$\Rightarrow y = f(x) = \dfrac{-1}{x-2} + 1$ 0,5 P

$$g(x) = |f(x)| = \left| \frac{-1}{x-2} + 1 \right|$$

Wertetabelle:

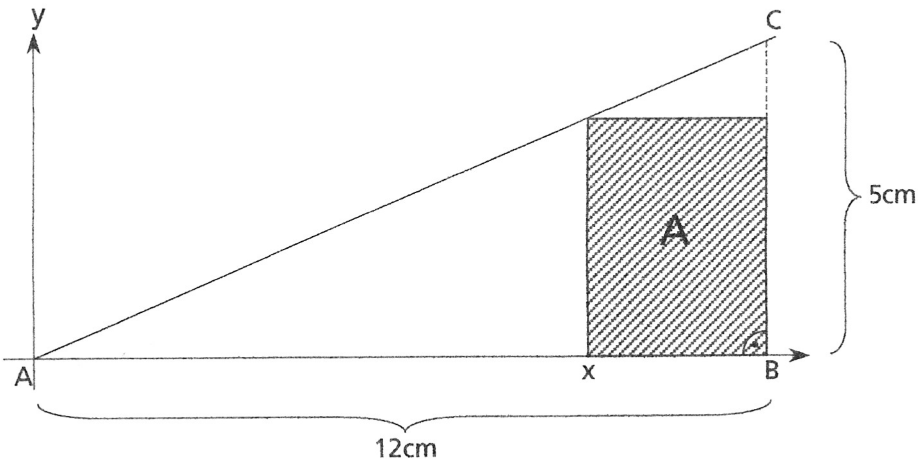
x	-2	-1	0	1	1,5	2	2,5	3	3,5	4	6
y	1,25	$1,\bar{3}$	1,5	2	3	VAS	1	0	$0,\bar{3}$	0,5	0,75

$2,1 \rightarrow 9$



3 P

5



a)

0,5P 0,5P

$$A = (12 - x) \cdot y = (12 - x) \cdot \frac{5x}{12}$$

$$\underline{\underline{A = -\frac{5x^2}{12} + 5x}}$$ 1P

b)

$$A = -\frac{5}{12}(x^2 - 12x)$$

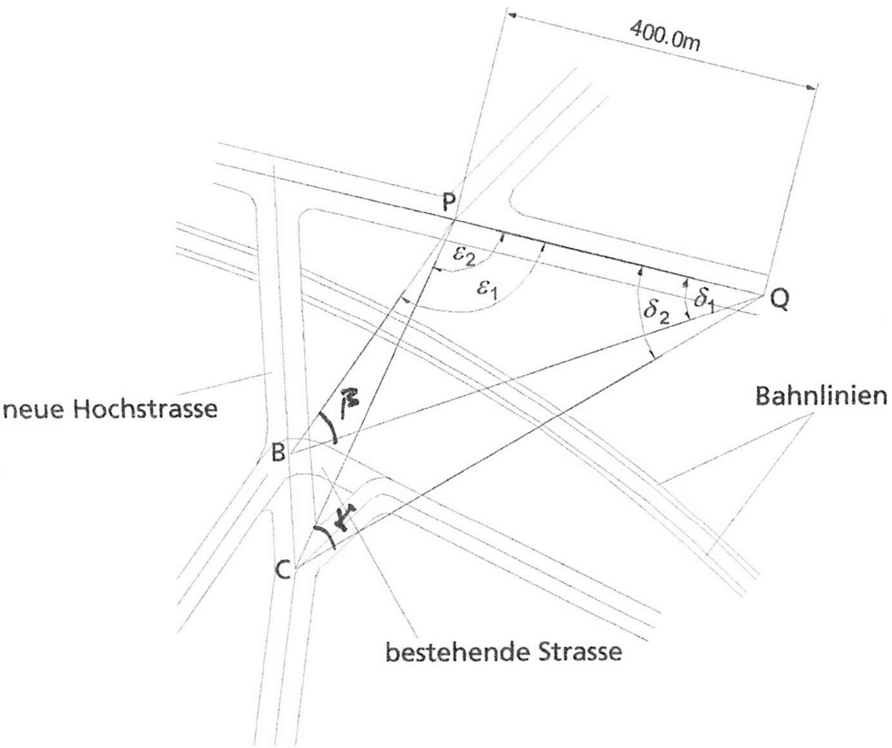
$$A = -\frac{5}{12}\left[(x-6)^2 - 36\right]$$

$$\underline{A = -\frac{5}{12}(x-6)^2 + 15}$$ 1P

$$\rightarrow \underline{\underline{A_{max} = 15}} \; ; \; \underline{\underline{x = 6}}$$

0,5P 0,5P

6



$$\beta = 180 - (\varepsilon_1 + \delta_1)$$
$$\beta = 180 - (114 + 33{,}5)$$
$$\underline{\beta = 32{,}5^\circ}$$ 0,5P

$$\gamma = 180 - (\varepsilon_2 + \delta_2)$$
$$\gamma = 180 - (109{,}2 + 46{,}3)$$
$$\underline{\gamma = 24{,}5^\circ}$$ 0,5P

<u>Sinussatz:</u>

$$\frac{\overline{PC}}{\sin 46{,}3^\circ} = \frac{\overline{PQ}}{\sin 24{,}5^\circ}$$

$$\overline{PC} = \overline{PQ}\,\frac{\sin 46{,}3^\circ}{\sin 24{,}5^\circ} = 400 \cdot \frac{\sin 46{,}3^\circ}{\sin 24{,}5^\circ}$$

$$\underline{\underline{\overline{PC} = 697{,}35\,m}}$$ 1P

$$\overline{PB} = \overline{PQ} \cdot \frac{\sin 33{,}5^\circ}{\sin 32{,}5^\circ} = 400 \cdot \frac{\sin 33{,}5^\circ}{\sin 32{,}5^\circ}$$

$$\underline{\underline{\overline{PB} = 410{,}90\,m}}$$ 1P

<u>Cosinussatz:</u>

$$\overline{BC}^2 = \overline{PB}^2 + \overline{PC}^2 - 2\,\overline{PB}\,\overline{PC}\cos(114 - 109{,}2)$$

$$\overline{BC}^2 = 410{,}90^2 + 697{,}35^2 - 2 \cdot 410{,}90 \cdot 697{,}35 \cdot \cos 4{,}8^\circ$$

$$\underline{\underline{\overline{BC} = 289{,}94\,m}}$$ 1P

• prop Abzüge

7a

$f(x) = a \cdot \cos(\omega x + \varphi) + d$

Aus dem Graphen:

$a = 1{,}2$ 0,5

$d = 1$ 0,5

$\varphi = 0$ → Spiegelung an der x-Achse!
0,5
→ (-1) 0,5

0,5
Periodizität $p = 2\pi \Rightarrow \underline{\omega = 1}$!

$\Rightarrow \quad \underline{\underline{f(x) = -1{,}2 \cos(x) + 1}}$ 0,5

oder $\underline{\underline{f(x) = 1{,}2 \cos(x + \pi) + 1}}$

7 b

$$\frac{\sin(x)}{1-\cos(x)} = \tan(x) \quad ; \quad x \in [0; 2\pi[$$

D: $\cos(x) = 1 \rightarrow x = 0$ 0,5 P

$\tan(x) \rightarrow x = \frac{\pi}{2}; \frac{3\pi}{2}$ 1 P

$\rightarrow \mathbb{D} =]0; 2\pi[\setminus \{\frac{\pi}{2}; \frac{3\pi}{2}\}$ 0,5 P

$$\frac{\sin(x)}{1-\cos(x)} = \frac{\sin(x)}{\cos(x)}$$ 0,5

$$\sin(x)\cos(x) = \sin(x) - \sin(x)\cos(x)$$

$$2\sin(x)\cos(x) = \sin(x)$$

$$\underbrace{\sin(x)}_{0;\,\pi;\,2\pi}\,[\underbrace{2\cos(x) - 1}_{=0}] = 0$$ 1 P

0,5 P 0,5 P

$\rightarrow \cos(x) = 0{,}5$

$x_3 = \cos^{-1}(0{,}5)$

$x_3 = \frac{\pi}{3}$ 1 P

$x_4 = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$\Rightarrow$ Lösungen:

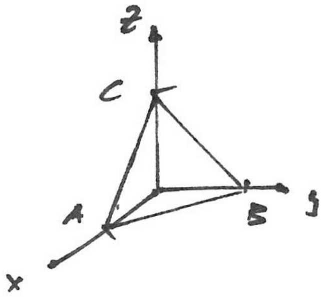
$$\mathbb{L} = \{\frac{\pi}{3}; \pi; \frac{5\pi}{3}\}$$ 0,5 P

8

$$G: \vec{r}_a = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} a \\ 1 \\ a+2 \end{pmatrix}$$

$A(10/0/0)$, $B(0/5/0)$, $C(0/0/5)$

a)



$$E: n_1 x + n_2 y + n_3 z = k \quad / :k$$

$$\frac{n_1 x}{k} + \frac{n_2 y}{k} + \frac{n_3 z}{k} = 1$$

$$\Rightarrow \frac{k}{n_1} = 10 \; , \; \frac{k}{n_2} = 5 \; , \; \frac{k}{n_3} = 5$$

$$\Rightarrow \frac{x}{10} + \frac{y}{5} + \frac{z}{5} = 1 \quad / \cdot 10$$

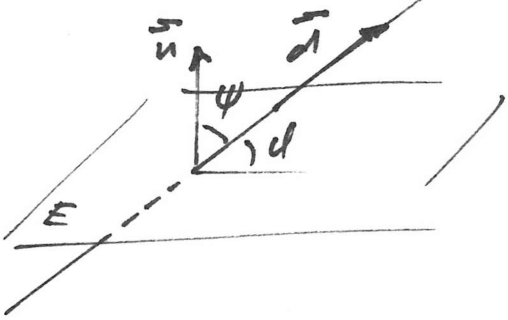
$$\underline{\underline{E: x + 2y + 2z = 10}} \qquad 2P$$

---

b)

$$G: \vec{r}_{-1} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \quad 0{,}5P \; , \; a = -1$$

$$E: x + 2y + 2z = 10 \Rightarrow \vec{n} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \quad 0{,}5P$$



$$\cos(\psi) = \sin(90 - \psi)$$
$$= \sin(\varphi)$$

$$\Rightarrow \sin(\varphi) = \frac{\vec{n} \cdot \vec{d}}{n \cdot d}$$

$$\Rightarrow \varphi = \arcsin \frac{\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}}{3 \cdot \sqrt{3}} \Rightarrow \frac{\cancel{3}}{\cancel{3}\sqrt{3}} \Rightarrow \frac{1}{\sqrt{3}}$$

$$\underline{\underline{\varphi}} = \arcsin\left(\frac{1}{\sqrt{3}}\right) = \underline{\underline{35{,}3°}} \qquad 1P$$

8c

$G: \quad a = -2$

$G: \vec{r}_2 = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \qquad P: \vec{r}_P = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}, \quad \vec{d} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

0,5

$E: \quad x_1 + 2x_2 + 2x_3 - 10 = 0$

0,5

$P: \quad 4 + 2 \cdot 2 + 2 \cdot 1 - 10 = 0 \quad \Rightarrow \quad P \in E!$

$\vec{n} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \quad \vec{d} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \qquad \underline{\vec{n} \cdot \vec{d} = 0}$

$\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = -2 + 2 + 0 = 0$

0,5

$\Rightarrow \underline{\underline{G \subset E}}$

$G_{AB}: \quad \vec{r} = \vec{r}_A + \mu \left(\vec{r}_B - \vec{r}_A \right)$

$\vec{r} = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 - 10 \\ 5 - 0 \\ 0 - 0 \end{pmatrix}$

$\vec{r} = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -10 \\ 5 \\ 0 \end{pmatrix} \quad \Rightarrow \quad \vec{m} = \begin{pmatrix} -10 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

0,5

<u>Parallelität</u> $\quad \vec{d} = \nu \vec{m}$

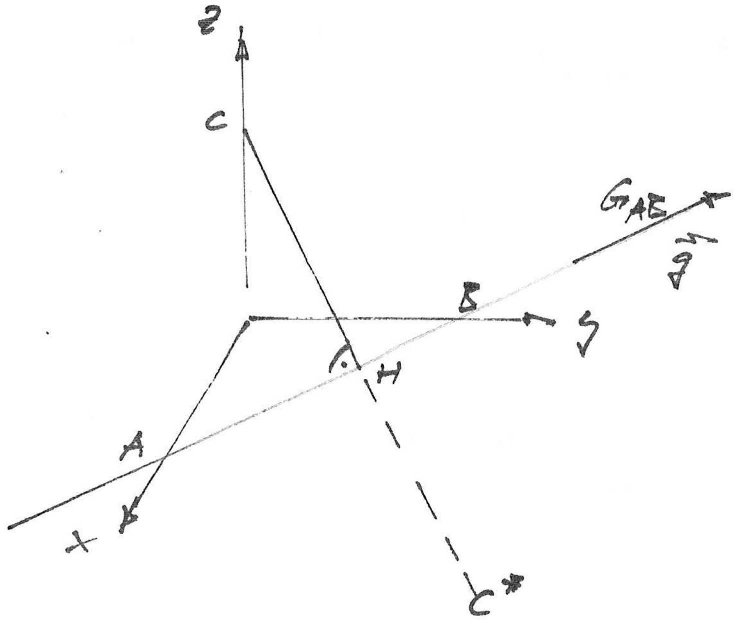
$\Rightarrow \underline{\underline{\vec{d} = \vec{m}}}$ 0,5

0,5

$A \notin G \quad \Rightarrow \quad \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \lambda \neq \text{Konst.}$

$\underline{\underline{\lambda \notin R}}$

8a1



$$G_{AB}: \quad \vec{r} = \begin{pmatrix}10\\0\\0\end{pmatrix} + \lambda \begin{pmatrix}-2\\1\\0\end{pmatrix} = \underline{\begin{pmatrix}10-2\lambda\\\lambda\\0\end{pmatrix}}$$

$$\vec{r} = \begin{pmatrix}x\\y\\z\end{pmatrix} \rightarrow \vec{r} = \vec{r}_H$$

$$\vec{HC} \cdot \vec{g} = 0$$

$$\left[\begin{pmatrix}0\\0\\5\end{pmatrix} - \begin{pmatrix}10-2\lambda\\\lambda\\0\end{pmatrix}\right]\begin{pmatrix}-2\\1\\0\end{pmatrix} = 0$$

$$\begin{pmatrix}2\lambda-10\\-\lambda\\5\end{pmatrix}\begin{pmatrix}-2\\1\\0\end{pmatrix} = -4\lambda + 20 - \lambda = 0$$

$$5\lambda = 20$$

$$\underline{\lambda = 4}$$

$$\underline{\vec{r}_H = \begin{pmatrix}2\\4\\0\end{pmatrix}}$$

$$\vec{CC^*} = 2 \cdot \vec{CH}$$

$$C^*: \quad \vec{r}_{C^*} = \vec{r}_C + 2\,\vec{CH} = \begin{pmatrix}0\\0\\5\end{pmatrix} + 2\begin{pmatrix}2-0\\4-0\\0-5\end{pmatrix} = \underline{\underline{\begin{pmatrix}4\\8\\-5\end{pmatrix}}}$$

__9a__

$$\vec{v} = \begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} a-1 \\ -2 \\ 1 \end{pmatrix}$$

$$A = |\vec{v} \times \vec{w}|$$

$$\Rightarrow \vec{v} \times \vec{w} = \begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} a-1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} 0 & -2 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} a & a-1 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} a & a-1 \\ 0 & -2 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -(a-a+1) \\ -2a \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -2a \end{pmatrix}$$ 7VP

$$A = \sqrt{2^2 + (-1)^2 + (-2a)^2}$$

$$A^2 = 4 + 1 + 4a^2 \qquad \underline{A = 3}$$

$$\Rightarrow 9 = 5 + 4a^2$$

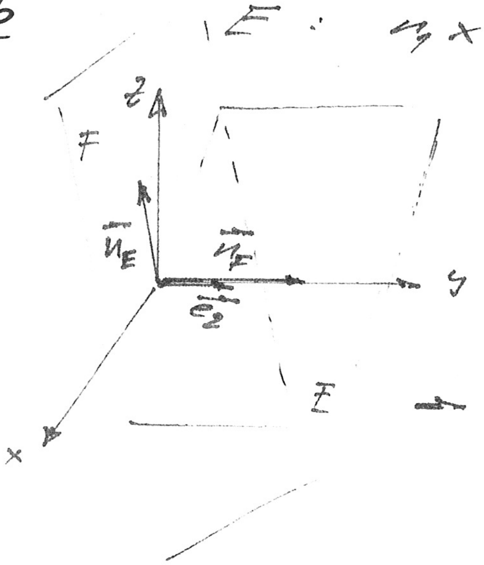
$$a^2 = 1$$

$$|a| = 1$$

$$\underline{\underline{a = \pm 1}}$$ 7VP

9b

$E: n_1 x + n_2 y + n_3 z = k$



$F: 2x - y + z = 5$

Gemäss Bedingungen:

$\underline{E: n_1 x + n_3 z = 0}$ 0,5 ✓

$\rightarrow \vec{n}_E \cdot \vec{n}_F = 0 \quad \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 0$ 0,5 ✓

$2n_1 - n_2 + n_3 = 0$

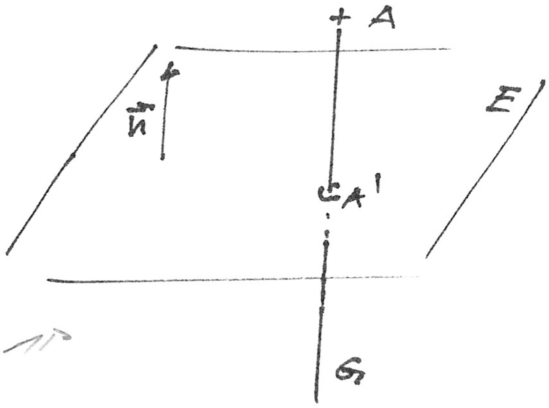
$\vec{n}_E \cdot \vec{e}_2 = 0 \quad \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0 \rightarrow \underline{n_2 = 0}\,!$ 0,5 ✓

$\rightarrow 2n_1 = -n_3$

Annahme: $n_1 = 1 \,;\, n_3 = -2 \rightarrow \underline{\underline{E: x - 2z = 0}}$

0,5 ✓

9c



$A(3|-2|5)$

$\vec{n} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ 0,5 ✓

$G: \vec{r} = \vec{r}_A + \lambda \vec{n}$

$G: \vec{r} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ 1 ✓

$E: x + 2y - z = 6$

$\rightarrow x = 3 + \lambda \,,\; y = -2 + 2\lambda \,,\; z = 5 - \lambda$

$\rightarrow 3 + \lambda + 2(-2 + 2\lambda) - 5 + \lambda = 6$

$\lambda + 4\lambda + \lambda + 3 - 4 - 5 = 6$

$6\lambda = 12 \rightarrow \underline{\lambda = 2}$ 1 ✓

$A' : \underline{\underline{\vec{r}_{A'}}} = \begin{pmatrix} 3+2 \\ -2+4 \\ 5-2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}}}$ 0,5 ✓