Aufgabe 1

$$\frac{3x+2}{x+3} - 1 \leq \frac{2x-6}{x-2}$$

$$\frac{3x+2-(x+3)}{x+3} \leq \frac{2x-6}{x-2}$$

$$\frac{3x+2-x-3}{x+3} \leq \frac{2x-6}{x-2}$$

$$\frac{2x-1}{x+3} \leq \frac{2x-6}{x-2}$$

$$\frac{2x-1}{x+3} - \frac{2x-6}{x-2} \leq 0$$

$$\frac{(2x-1)(x-2)-(2x-6)(x+3)}{(x+3)(x-2)} \leq 0$$

$$\frac{2x^2-5x+2-2x^2+18}{(x+3)(x-2)} \leq 0$$

$$\frac{-5x+20}{(x+3)(x-2)} \leq 0$$

I) $-5x+20 \geq 0 \quad , \quad -5x \geq -20 \quad , \quad x \leq 4$

a) $x+3 > 0 \quad \wedge \quad x-2 < 0$

$x > -3 \qquad x < 2 \qquad L_{Ia} = \{x \in \mathbb{R} \mid -3 < x < 2\}$

b) $x+3 < 0 \quad \wedge \quad x-2 > 0$

$x < -3 \qquad x > 2 \qquad L_{Ib} = \{\}$

II) $-5x+20 \leq 0 \quad , \quad -5x \leq -20 \quad , \quad x \geq 4$

a) $x+3 > 0 \quad \wedge \quad x-2 > 0$

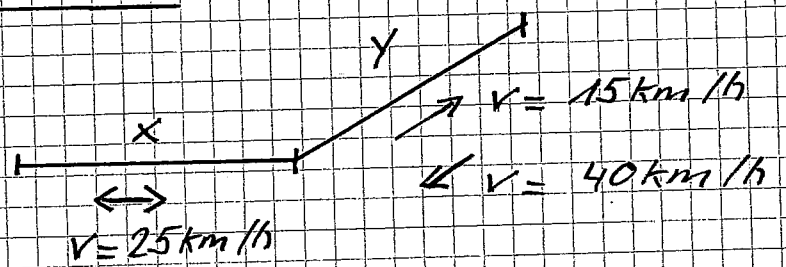
$x > -3 \qquad x > 2 \qquad L_{IIa} = \{x \in \mathbb{R} \mid x \geq 4\}$

b) $x+3 < 0 \quad \wedge \quad x-2 < 0$

$x < -3 \qquad x < 2 \qquad L_{IIb} = \{\}$

$$L = \{x \in \mathbb{R} \mid -3 < x < 2 \quad \vee \quad x \geq 4\}$$

Aufgabe 2



x: Flachstrecke (km)

y: Bergstrecke (km)

a) $s = v \cdot t \qquad t = \frac{s}{v}$

① $\frac{x}{25} + \frac{y}{15} = 4\frac{1}{3}$

② $\frac{x}{25} + \frac{y}{40} = 3$

$$\frac{x}{25} + \frac{y}{15} = 4\frac{1}{3}$$
$$-\frac{x}{25} - \frac{y}{40} = -3$$

$$\frac{5y}{120} = \frac{4}{3}$$

$$y = \frac{4 \cdot 120}{3 \cdot 5} = \underline{\underline{32\,km}}$$

$$\frac{x}{25} + \frac{32}{40} = 3$$

$$\frac{x}{25} = \frac{11}{5}$$

$$x = 55\,km$$

b) gesamte Tour:

$$2x + 2y = 110\,km + 64\,km = \underline{\underline{174\,km}}$$

Aufgabe 3

$$\left(\frac{4s}{s-2} - \frac{7s-15}{2s-7}\right) \cdot \frac{3(s+4)}{s+6} = 0$$

$$D = \mathbb{R} \setminus \left\{-6; 2; \frac{7}{2}\right\}$$

I)
$$\frac{4s}{s-2} - \frac{7s-15}{2s-7} = 0$$

$$4s(2s-7) - (7s-15)\cdot(s-2) = 0$$

$$8s^2 - 28s - 7s^2 + 29s - 30 = 0$$

$$s^2 + s - 30 = 0$$

$$s_{1/2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-30)}}{2 \cdot 1}$$

$$= \frac{-1 \pm 11}{2}$$

$$s_1 = \frac{-1+11}{2} = 5$$

$$s_2 = \frac{-1-11}{2} = -6$$

II)
$$\frac{3(s+4)}{s+6} = 0$$

$$s+4 = 0$$

$$s_3 = -4$$

$$L = \{-4; 5\}$$

Aufgabe 4

$$W_n = W_0 \cdot \left(1 - \frac{p}{100}\right)^n$$

a) $\frac{1}{4} W_0 = W_0 \cdot \left(1 - \frac{3}{100}\right)^n$

$0{,}25 = 0{,}97^n$

$\lg 0{,}25 = \lg 0{,}97^n$

$\lg 0{,}25 = n \cdot \lg 0{,}97$

$$n = \frac{\lg 0{,}25}{\lg 0{,}97} = 45{,}51 \text{ Zeiteinheiten}$$

$\hat{=}$ **227,57 Std**

b) $W_0 \cdot \left(1 - \frac{3}{100}\right)^{\frac{24}{5}} = W_0 \cdot \left(1 - \frac{x}{100}\right)$

$0{,}97^{\frac{24}{5}} = 1 - \frac{x}{100}$

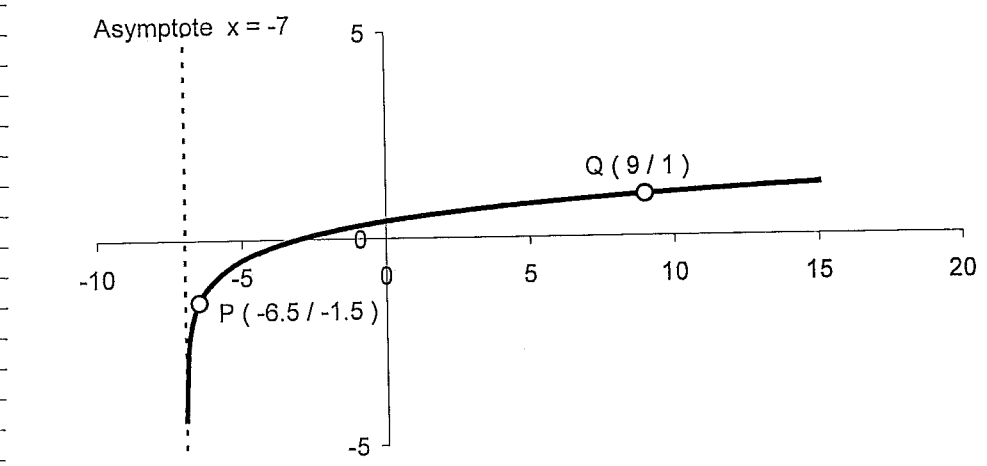
$\frac{x}{100} = 1 - 0{,}97^{\frac{24}{5}}$

$x = 100 \cdot \left(1 - 0{,}97^{\frac{24}{5}}\right)$

$x = 13{,}6\ \%$

x: prozentualer Energieverlust pro Tag

Aufgabe 5



$y = \log_a(x+b) + c$

$b = 7$

$\longrightarrow y = \log_a(x+7) + c$

P.: $-1{,}5 = \log_a(-6{,}5+7) + c$

$-1{,}5 = \log_a 0{,}5 + c$

$\longrightarrow c = -1{,}5 - \log_a 0{,}5$

Q.: $1 = \log_a(9+7) + c$

$1 = \log_a 16 + c$

$\longrightarrow c = 1 - \log_a 16$

$-1{,}5 - \log_a 0{,}5 = 1 - \log_a 16$

$\log_a 16 - \log_a 0{,}5 = 2{,}5$

$\log_a \frac{16}{0{,}5} = 2{,}5$

$\log_a 32 = 2{,}5$

$a^{2{,}5} = 32$

$a = 32^{\frac{1}{2{,}5}}$

$a = 4$

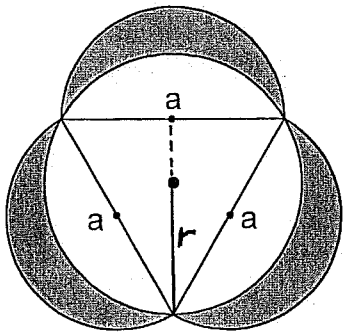
$c = 1 - \log_4 16$

$c = 1 - 2$

$c = -1$

$y = \log_4(x+7) - 1$

Aufgabe 6



$$r = \frac{2}{3}h = \frac{2}{3} \cdot \frac{a}{2} \cdot \sqrt{3} = \frac{a \cdot \sqrt{3}}{3}$$

$$A = \frac{a^2 \cdot \sqrt{3}}{4} + 3 \cdot \frac{a^2 \pi}{4 \cdot 2} - \left(\frac{a \cdot \sqrt{3}}{3}\right)^2 \cdot \pi$$

$$= \frac{a^2 \sqrt{3}}{4} + \frac{3a^2 \pi}{8} - \frac{a^2 \pi}{3}$$

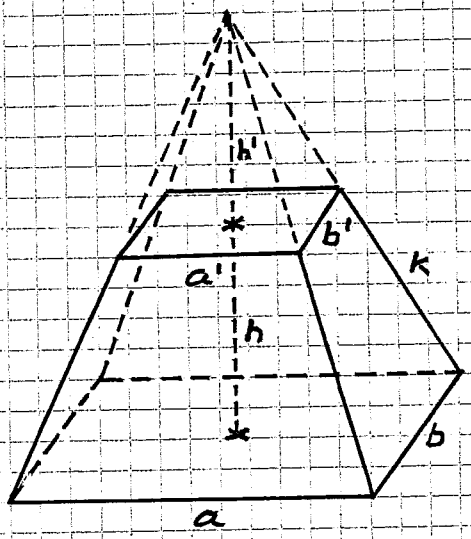
$$= \frac{a^2 \cdot 6 \cdot \sqrt{3}}{24} + \frac{9a^2 \pi}{24} - \frac{8a^2 \pi}{24}$$

$$= \frac{a^2 \cdot 6 \cdot \sqrt{3}}{24} + \frac{a^2 \pi}{24}$$

$$= a^2 \frac{6 \cdot \sqrt{3} + \pi}{24}$$

$$= a^2 \cdot 0{,}5639$$

Aufgabe 7



a) $A_G = k^2 \cdot A_D$

$k^2 = 2{,}25$

$k = \sqrt{2{,}25} = \underline{1{,}5}$

$a = 1{,}5 \cdot a' \longrightarrow a' = \frac{96}{1{,}5} = \underline{64\,cm}$

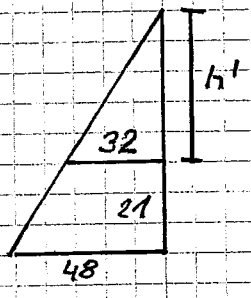
$b = 1{,}5 \cdot b' \longrightarrow b' = \frac{72}{1{,}5} = \underline{48\,cm}$

b) $d_G = \sqrt{96^2 + 72^2} = 120\,cm$

$d_D = \sqrt{64^2 + 48^2} = 80\,cm$

$h = \sqrt{29^2 - \left(\frac{120-80}{2}\right)^2} = \sqrt{29^2 - 20^2} = \underline{21\,cm}$

c)



$h' : 32 = (h' + 21) : 48$

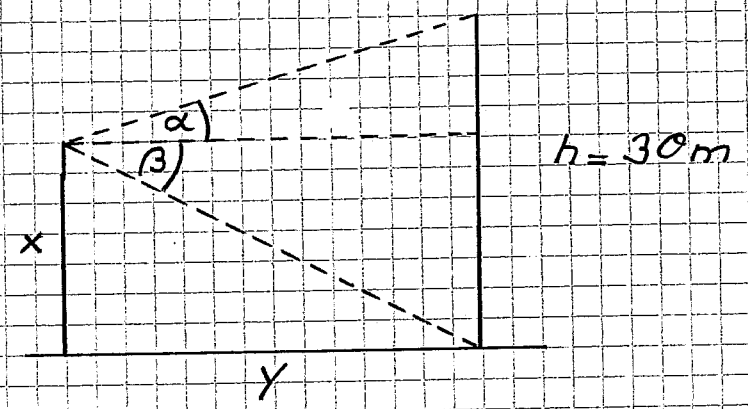
$48 \cdot h' = 32 \cdot (h' + 21)$

$48\,h' = 32\,h' + 672$

$16\,h' = 672$

$\underline{h' = 42\,cm}$

Aufgabe 8



① $\frac{x}{y} = \tan\beta \longrightarrow y = \frac{x}{\tan\beta}$

② $\frac{h-x}{y} = \tan\alpha \longrightarrow y = \frac{h-x}{\tan\alpha}$

$$\frac{x}{\tan\beta} = \frac{h-x}{\tan\alpha}$$

$$x \cdot \tan\alpha = (h-x) \cdot \tan\beta$$

$$x \cdot \tan\alpha = h \cdot \tan\beta - x \cdot \tan\beta$$

$$x \cdot \tan\alpha + x \cdot \tan\beta = h \cdot \tan\beta$$

$$x(\tan\alpha + \tan\beta) = h \cdot \tan\beta$$

$$x = \frac{h \cdot \tan\beta}{\tan\alpha + \tan\beta}$$

$$\underline{\underline{x = 17{,}4\,m}}$$

$$y = \frac{\frac{h \cdot \tan\beta}{\tan\alpha + \tan\beta}}{\tan\beta}$$

$$y = \frac{h}{\tan\alpha + \tan\beta}$$

$$\underline{\underline{y = 50{,}53\,m}}$$

Aufgabe 9

$$\frac{\sin 2\alpha}{4} + \sin^2\alpha = 1$$

$$\frac{2\sin\alpha\cos\alpha}{4} = 1 - \sin^2\alpha$$

$$\frac{\sin\alpha\cos\alpha}{2} = \cos^2\alpha$$

$$0 = \cos^2\alpha - \frac{\sin\alpha\cos\alpha}{2}$$

$$0 = \cos\alpha \cdot \left(\cos\alpha - \frac{\sin\alpha}{2}\right)$$

$$0 = \cos\alpha \cdot (2\cos\alpha - \sin\alpha)$$

I) $\cos\alpha = 0 \longrightarrow \alpha_1 = 90°$

$\alpha_2 = 270°$

II) $2\cos\alpha - \sin\alpha = 0$

$2\cos\alpha = \sin\alpha$

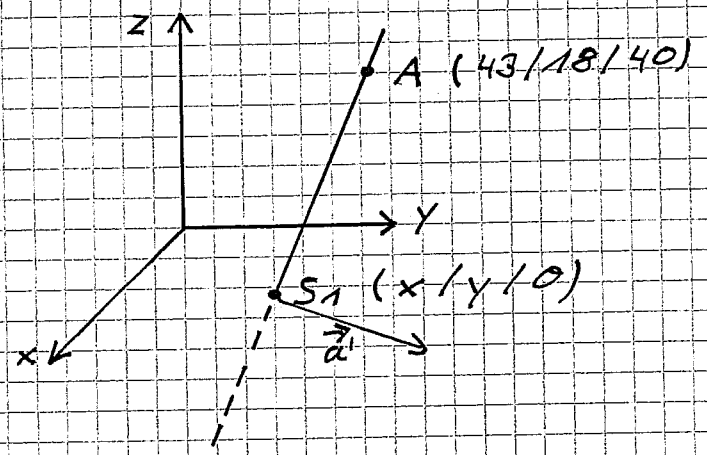
$\frac{\sin\alpha}{\cos\alpha} = 2$

$\tan\alpha = 2 \longrightarrow \alpha_3 = 63{,}43°$

$\alpha_4 = 243{,}43°$

$$L = \{63{,}43°;\ 90°;\ 243{,}43°;\ 270°\}$$

Aufgabe 10



a)

$$\begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} 43 \\ 18 \\ 40 \end{pmatrix} + t \cdot \begin{pmatrix} 7 \\ -3 \\ 5 \end{pmatrix}$$

$x = 43 + 7t$

$y = 18 - 3t$

$0 = 40 + 5t \longrightarrow t = -\frac{40}{5} = -8$

$\longrightarrow x = 43 - 7 \cdot 8 = 43 - 56 = -13$

$y = 18 + 3 \cdot 8 = 18 + 24 = 42$

$\longrightarrow \underline{\underline{S_1 (-13 / 42 / 0)}}$

b) $\vec{a} = \begin{pmatrix} 7 \\ -3 \\ 5 \end{pmatrix}$, $\vec{a}' = \begin{pmatrix} 7 \\ -3 \\ 0 \end{pmatrix}$

$$\cos\alpha = \frac{\vec{a} \cdot \vec{a}'}{|\vec{a}| \cdot |\vec{a}'|}$$

$$= \frac{\begin{pmatrix} 7 \\ -3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 7 \\ -3 \\ 0 \end{pmatrix}}{\sqrt{7^2 + (-3)^2 + 5^2} \cdot \sqrt{7^2 + (-3)^2 + 0^2}} = \frac{7 \cdot 7 + (-3) \cdot (-3) + 5 \cdot 0}{\sqrt{83} \cdot \sqrt{58}}$$

$$= \frac{49 + 9}{\sqrt{83} \cdot \sqrt{58}} = \frac{58}{\sqrt{83} \cdot \sqrt{58}} = 0,8359$$

$\underline{\underline{\alpha = 33,29°}}$