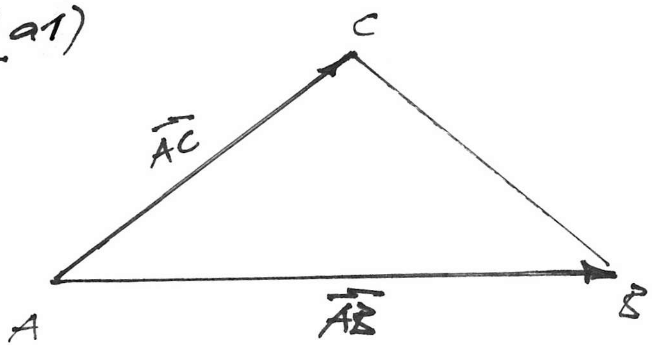
1 a1)



$A(m|3|m)$

$B(8|0|m)$

$C(-2|5|m)$

$$A = \frac{1}{2} |\vec{AC} \times \vec{AB}| = 2{,}5$$

$$\vec{AC} = \vec{r_C} - \vec{r_A} = \begin{pmatrix} -2-m \\ 5-3 \\ m-m \end{pmatrix} = \begin{pmatrix} -2-m \\ 2 \\ 0 \end{pmatrix}$$ 0,5 P

$$\vec{AB} = \vec{r_B} - \vec{r_A} = \begin{pmatrix} 8-m \\ 0-3 \\ m-m \end{pmatrix} = \begin{pmatrix} 8-m \\ -3 \\ 0 \end{pmatrix}$$ 0,5 P

$$\vec{AC} \times \vec{AB} = \begin{pmatrix} -2-m \\ 2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 8-m \\ -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \begin{vmatrix} -2-m & 8-m \\ 2 & -3 \end{vmatrix} \end{pmatrix}$$

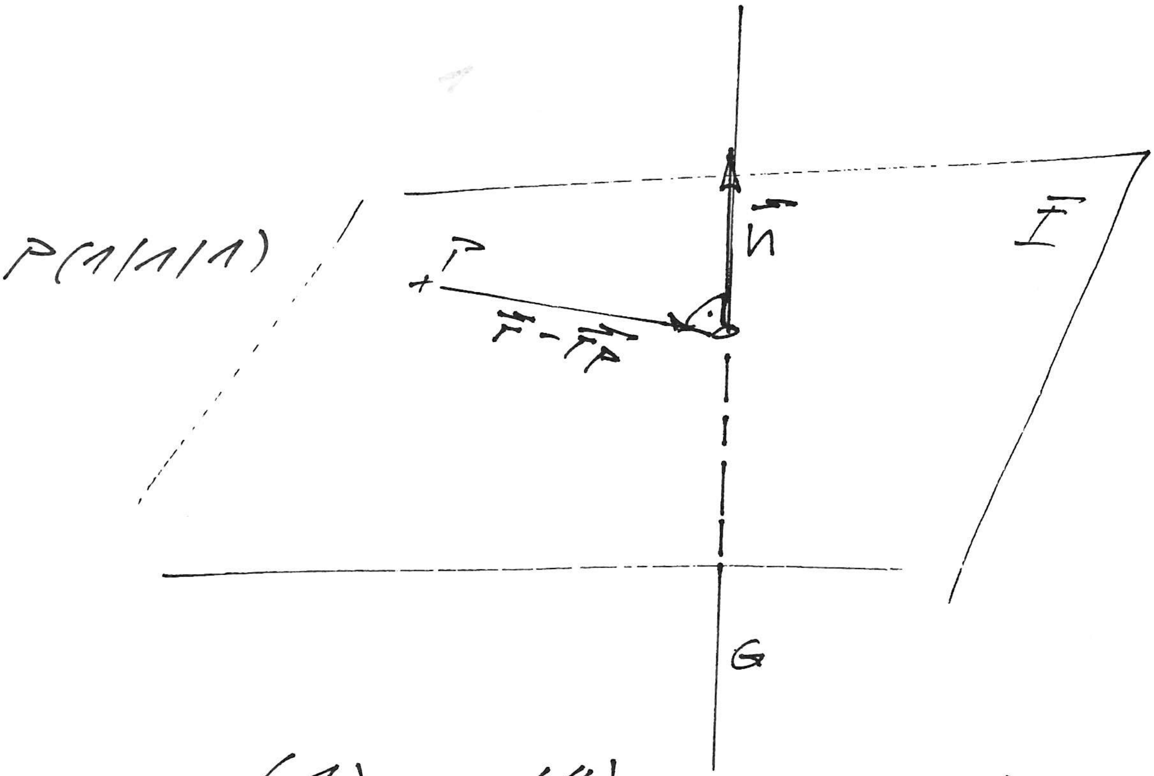
$$\vec{AC} \times \vec{AB} = \begin{pmatrix} 0 \\ 0 \\ 6+3m-16+2m \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0 \\ 0 \\ 5m-10 \end{pmatrix}}}$$ 1 P

a2) $|\vec{AC} \times \vec{AB}| = \sqrt{(5m-10)^2} = |5m-10|$

$= \underline{\pm(5m-10)}$ 1 P

$\Rightarrow \frac{1}{2}(5m-10) = 2{,}5$	$\frac{1}{2}(-5m+10) = 2{,}5$
$5m-10 = 5$	$-5m+10 = 5$
$5m = 15$	$\underline{\underline{m = 1}}$
$\underline{\underline{m = 3}}$	0,5 P
0,5 P	

1b



$$G: \quad \vec{r} = \begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \qquad \vec{n} = \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$$

$$E: \quad \vec{n}\,(\vec{r} - \vec{r}_P) = 0$$

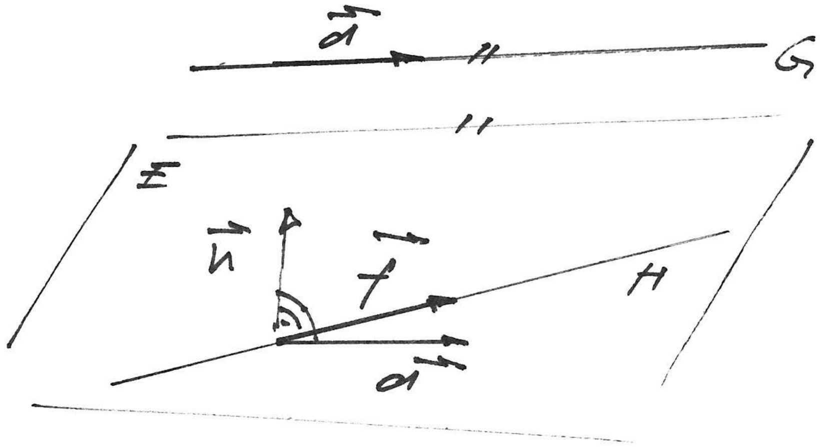
$$E: \quad \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \left[\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right] = 0$$

$$E: \quad \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \left[\begin{pmatrix} x-1 \\ y-1 \\ z-1 \end{pmatrix} \right] = 0$$

$$4x - 4 + 3y - 3 - z + 1 = 0$$

$$\underline{\underline{E: \quad 4x + 3y - z = 6}}$$

1c



$$\vec r_P = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$$

$$G: \vec r = \begin{pmatrix} -3+\lambda \\ 1+2\lambda \\ 5-2\lambda \end{pmatrix} \qquad H: \vec r = \begin{pmatrix} 2+3\mu \\ -3+\mu \\ 4+\mu \end{pmatrix}$$

$$\rightarrow \quad \vec d = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$ 2VP

$$\vec f = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$$ 2VP

$$\underline{\vec n} = \vec d \times \vec f = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} = \underline{\begin{pmatrix} 4 \\ -7 \\ -5 \end{pmatrix}}$$ 1P

$$E: \quad \vec n \cdot (\vec r - \vec r_P) = 0$$

$$\begin{pmatrix} 4 \\ -7 \\ -5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 4 \\ -7 \\ -5 \end{pmatrix}\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} = 0$$ 1P

$$\underline{\underline{E: \quad 4x - 7y - 5z = 9}}$$ 1P

2

a) $p: \vec{r} = \vec{r}_A + \lambda(\vec{r}_B - \vec{r}_A)$

$$\vec{r} = \begin{pmatrix}6\\4\\5\end{pmatrix} + \lambda\begin{pmatrix}3-6\\0-4\\4-5\end{pmatrix} = \begin{pmatrix}6\\4\\5\end{pmatrix} + \lambda\begin{pmatrix}-3\\-4\\-1\end{pmatrix}$$

$$\underline{p: \vec{r} = \begin{pmatrix}6\\4\\5\end{pmatrix} + \lambda\begin{pmatrix}3\\4\\1\end{pmatrix}}$$ 1P

$\underline{s}: \vec{r} = \vec{r}_P + \mu\begin{pmatrix}0\\1\\1\end{pmatrix} = \underline{\begin{pmatrix}0\\-7\\0\end{pmatrix} + \mu\begin{pmatrix}0\\1\\1\end{pmatrix}}$ 0,5P

<u>Schnittpunkt T:</u>

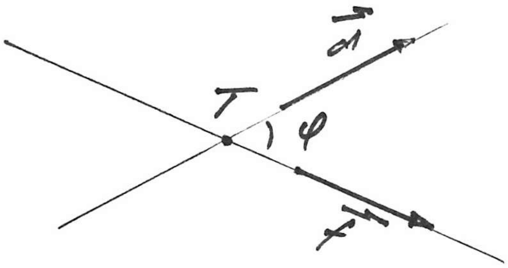
$$\left|\begin{array}{l} 6 + 3\lambda = 0 \\ 4 + 4\lambda = -7 + \mu \\ 5 + \lambda = 0 + \mu \end{array}\right| \Rightarrow \left|\begin{array}{l} 3\lambda = -6 \\ 4\lambda - \mu = -11 \\ \lambda - \mu = -5 \end{array}\right|$$

<u>2×2</u>

$$\left|\begin{array}{l} 4\lambda - \mu = -11 \\ \lambda - \mu = -5 \end{array}\right| \quad \underline{\lambda = -2} \quad \underline{\mu = 3}$$ 1P

$$\underline{\lambda = -2}$$ 0,5P

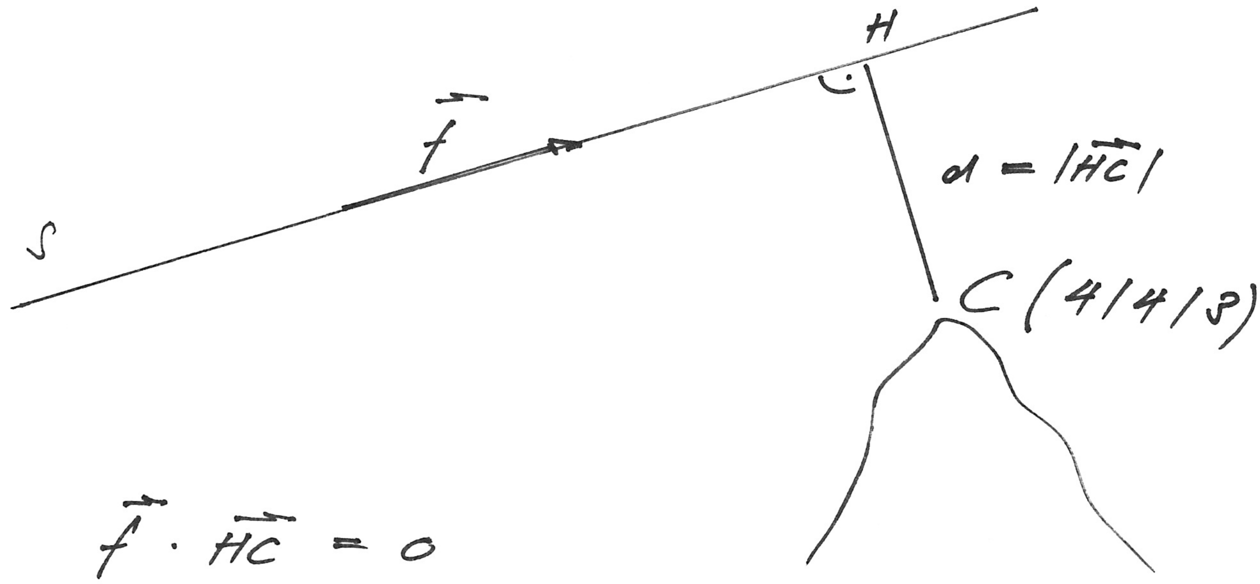
$\Rightarrow T: \underline{\underline{\vec{r}_T}} = \begin{pmatrix}0\\-7\\0\end{pmatrix} + 3\begin{pmatrix}0\\1\\1\end{pmatrix} = \underline{\underline{\begin{pmatrix}0\\-4\\3\end{pmatrix}}}$ 1P



$$\cos\varphi = \frac{\vec{d}\cdot\vec{f}}{|\vec{d}|\cdot|\vec{f}|} = \frac{\begin{pmatrix}3\\4\\1\end{pmatrix}\cdot\begin{pmatrix}0\\1\\1\end{pmatrix}}{\sqrt{26}\cdot\sqrt{2}}$$

$$\cos\varphi = \frac{5}{\sqrt{26}\cdot\sqrt{2}} = 0{,}698$$

$$\underline{\underline{\varphi = 46{,}1^\circ}}$$ 2P



$$\vec{f} \cdot \vec{HC} = 0$$

$$\vec{f} \cdot (\vec{r}_C - \vec{r}_H) = 0 \quad ; \quad \vec{r}_H = \begin{pmatrix} 0 \\ -7 \\ 0 \end{pmatrix} + \mu_H \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

0,5P 0,5P

$$\vec{f} \cdot \left[\vec{r}_C - \begin{pmatrix} 0 \\ -7 \\ 0 \end{pmatrix} - \mu_H \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right] = 0$$

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \begin{bmatrix} 4 - 0 - \mu_H \cdot 0 \\ 4 + 7 - \mu_H \\ 3 - 0 - \mu_H \end{bmatrix} = 0 \Rightarrow \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 11 - \mu_H \\ 3 - \mu_H \end{pmatrix} = 0$$

0,5 0,5P

$$\Rightarrow 11 - \mu_H + 3 - \mu_H = 0$$

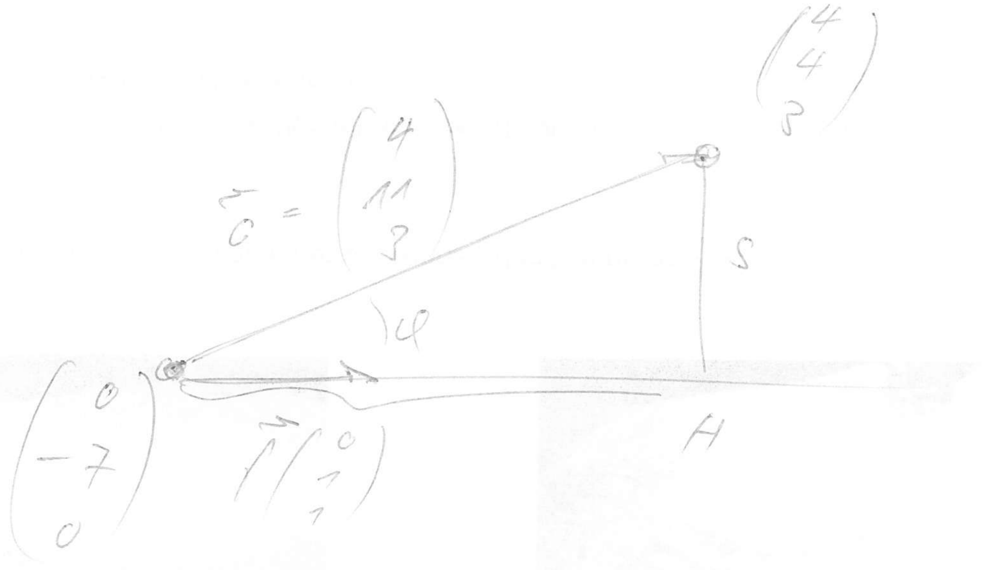
$$\underline{\mu_H = 7}$$ 0,5P

$$\underline{\underline{\vec{r}_H = \begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix}}}$$ 0,5P

$$\vec{HC} = \vec{r}_C - \vec{r}_H = \begin{pmatrix} 4 \\ 4 \\ 3 - 7 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -4 \end{pmatrix}$$ 0,5P

$$\underline{\underline{d}} = |\vec{HC}| = \sqrt{3 \cdot 16} = \underline{\underline{4 \cdot \sqrt{3}\ LE}}$$

6,928 LE 0,5P



$$\frac{\vec a \cdot \vec f = a \cdot f \cdot \cos\varphi}{f}$$

$$\frac{\vec a \cdot \vec f}{f} = a \cdot \cos\varphi \;\Big|\cdot \frac{\vec f}{f}$$

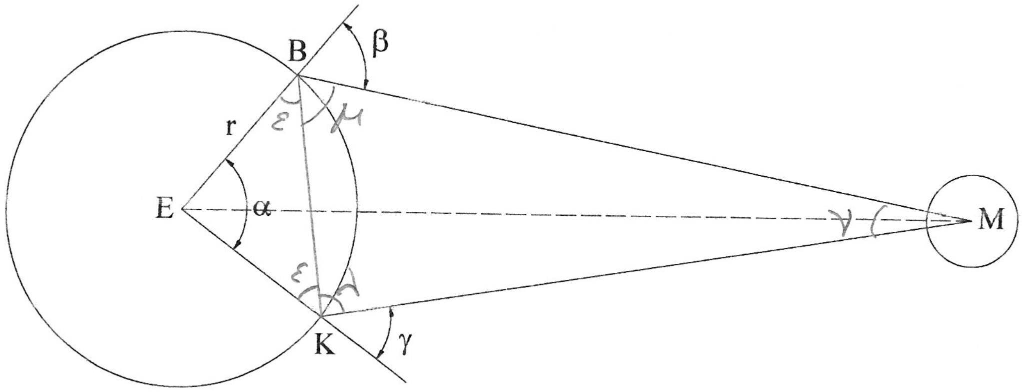
$$\boxed{\vec p = \frac{\vec a \cdot \vec f}{f^2}\,\vec f}$$

$$\frac{\begin{pmatrix}4\\11\\3\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix}}{2}\begin{pmatrix}0\\1\\1\end{pmatrix}$$

$$\Rightarrow H = \begin{pmatrix}0\\-7\\0\end{pmatrix} + \begin{pmatrix}0\\7\\7\end{pmatrix} = \begin{pmatrix}0\\0\\7\end{pmatrix}$$

$$\frac{14}{2}\cdot\begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}0\\7\\7\end{pmatrix}$$

3



$$\varepsilon = \frac{180° - \alpha}{2} = \frac{180° - 86{,}4411°}{2} = \underline{46{,}7795°}$$ 0,5

$$\overline{BK}^2 = 2r^2 - 2r^2\cos(\alpha) = 2\cdot 6370^2 - 2\cdot 6370^2\cdot\cos(86{,}4411°)$$

$$\overline{BK}^2 = 7{,}611\cdot 10^7\,\text{km}^2 \Rightarrow \underline{\overline{BK} = 8724{,}46\,\text{km}}$$ 1,5P

$$\underline{\mu} = 180° - \beta - \varepsilon = 180° - 41{,}2622 - 46{,}7795 = \underline{91{,}9584°}$$ 0,5P

$$\underline{\lambda} = 180° - \gamma - \varepsilon = 180° - 46{,}5603 - 46{,}7795 = \underline{86{,}6603°}$$ 0,5P

$$\underline{\nu} = 180° - \mu - \lambda = 180° - 91{,}9584 - 86{,}6603 = \underline{1{,}3814°}$$ 0,5P

$$\frac{\overline{BM}}{\sin(\lambda)} = \frac{\overline{BK}}{\sin(\nu)} \Rightarrow \overline{BM} = \overline{BK}\cdot\frac{\sin(\lambda)}{\sin(\nu)} = 8724{,}46\cdot\frac{\sin(86{,}6603)}{\sin(1{,}3814)}$$

$$\underline{\underline{\overline{BM} = 361'281\,\text{km}}}$$ 1P

$$\overline{EM}^2 = r^2 + \overline{BM}^2 - 2r\overline{BM}\cdot\cos(\varepsilon + \mu)$$

$$\overline{EM}^2 = 6370^2 + 361'281^2 - 2\cdot 6370\cdot 361'281\cdot\cos(138{,}738)$$

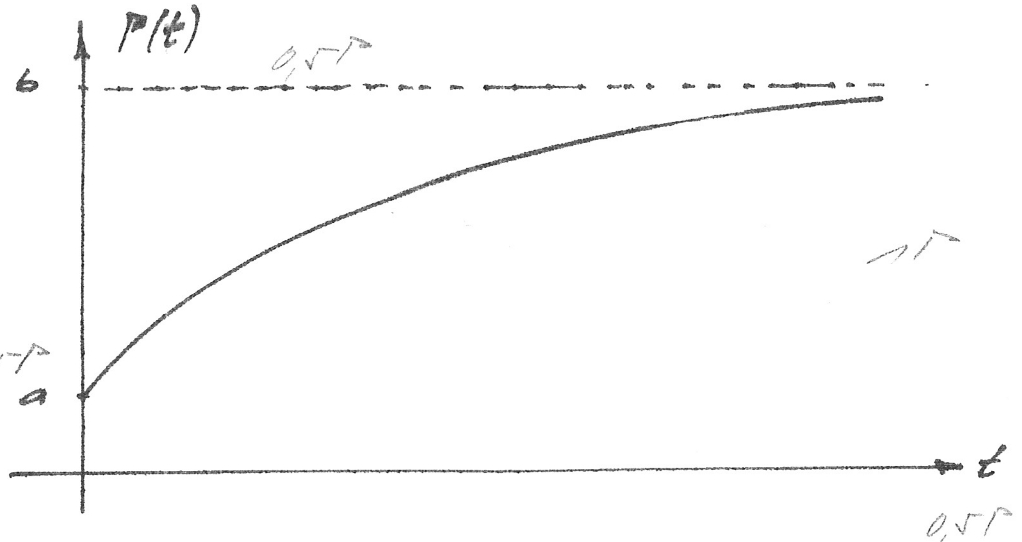
$$\overline{EM}^2 = 1{,}340\cdot 10^{11}\,\text{km}^2$$

$$\underline{\underline{\overline{EM} = 366'094\,\text{km}}}$$ 1,5P

4

$$P(t) = b - (b-a)e^{-\lambda t}$$

a)



0,5P

b)

b1) $P(5) = 350 \quad , \quad b = 1000 \; , \quad a = 200$

$$\rightarrow \quad 350 = 1000 - (1000 - 200)e^{-5\lambda}$$

$$800\, e^{-5\lambda} = 650$$

$$e^{-5\lambda} = \frac{\overset{13}{\cancel{650}}}{\underset{16}{\cancel{800}}} \quad / \ln()$$

$$-5\lambda = \ln\left(\frac{13}{16}\right)$$

$$\underline{\underline{\lambda}} = \frac{-1}{5} \cdot \ln\left(\frac{13}{16}\right) = \underline{\underline{0{,}0415}} \quad 2{,}5P$$

b2) $P = b - (b-a)e^{-\lambda t}$

$$(b-a)e^{-\lambda t} = b - P$$

$$\left(\frac{b-a}{b-P}\right) = e^{\lambda t} \quad / \ln()$$

$$\lambda t = \ln\left(\frac{b-a}{b-P}\right) \quad 1{,}5P$$

$$t = \frac{1}{\lambda} \cdot \ln\left(\frac{b-a}{b-P}\right) = \frac{1}{0{,}0415} \cdot \ln\left(\frac{\overset{2}{\cancel{1000-200}}}{\underset{1}{\cancel{1000-600}}}\right)$$

$$\underline{\underline{t = 16{,}69a}} \quad 0{,}5P$$

5c

$$\frac{2 - 2\cos^2(x)}{3\sin(2x)}$$

$$= \frac{2(\overbrace{1-\cos^2(x)}^{\sin^2(x)})}{3\cdot\underbrace{\sin(2x)}_{2\sin(x)\cos(x)}}$$

$$= \frac{\cancel{2}\cdot\sin^{\cancel{2}}(x)}{3\cdot\cancel{2}\,\cancel{\sin(x)}\cos(x)}$$

$$= \frac{1}{3}\,\frac{\sin(x)}{\cos(x)} = \underline{\underline{\frac{1}{3}\tan(x)}}$$

5d

$\sin^2(x) = a$ $\qquad 0 < a < 1$

$x \in [0; 2\pi[$

$|\sin(x)| = \sqrt{a}$

$\sin(x_1) = \sqrt{a} \quad \rightarrow \quad x_1 = \arcsin(\sqrt{a})$

$\sin(x_2) = -\sqrt{a} \quad \rightarrow \quad x_2 = -\arcsin(\sqrt{a})$

15P

Lösungen

$x_1 = \arcsin(\sqrt{a})$

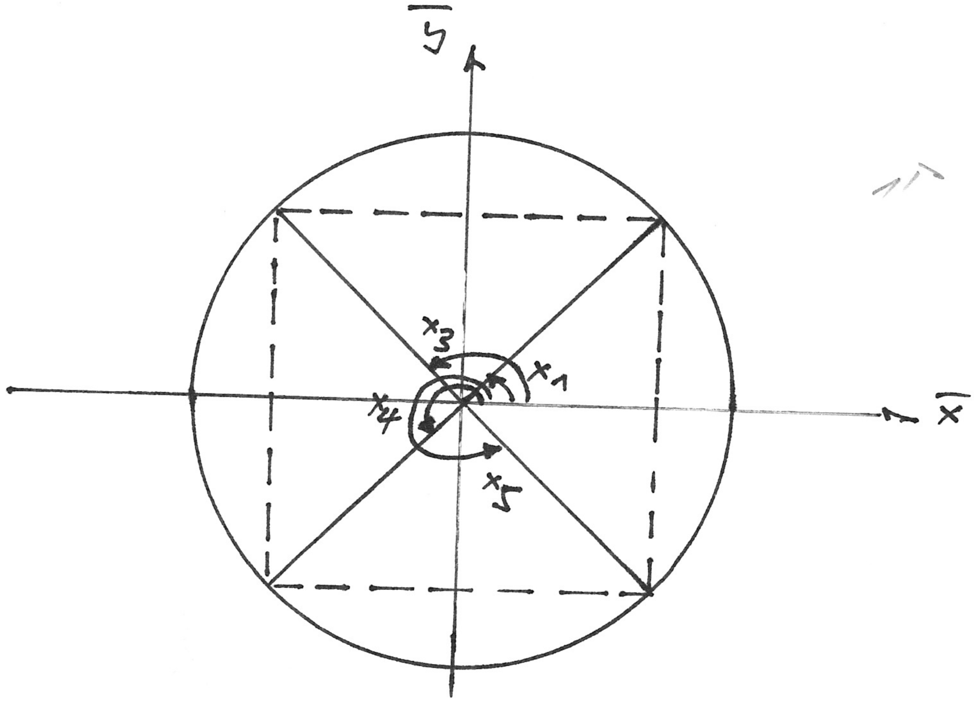
$x_3 = \pi - \arcsin(\sqrt{a})$

15P

$x_2 = -\arcsin(\sqrt{a})$

$\Rightarrow \quad x_4 = \pi - (-\arcsin(\sqrt{a})) = \pi + \arcsin(\sqrt{a})$

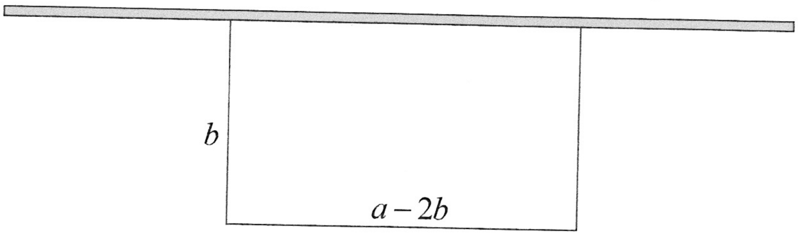
$x_5 = 2\pi + (-\arcsin(\sqrt{a})) = 2\pi - \arcsin(\sqrt{a})$



1P

5e

Rechteckiges Spielfeld an Lärmschutzwand



$A = b(a-2b) = ab - 2b^2$ 1P

$a \le 30$ 0,5

$\Rightarrow \quad A = -2b^2 + 30b$ 1P

$A = f(b) = -2(b^2 - 15b)$ 0,5

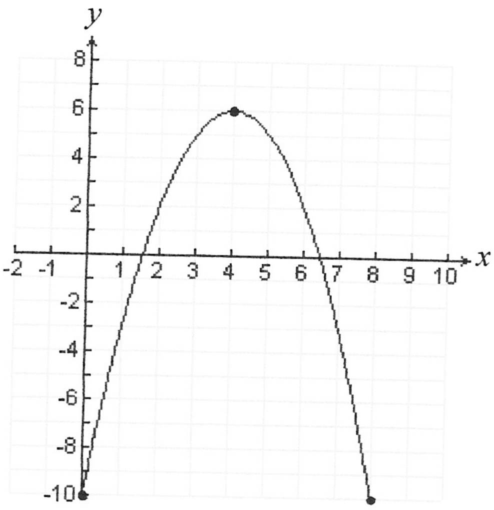
$= -2(b - 7{,}5)^2 + 112{,}5$ 1P

$A_{max} = 112{,}5 \text{ m}^2$

$\underline{\underline{b = 7{,}5 \text{ m}}}$ 0,5

$a - 2b = 30 - 15 = \underline{\underline{15 \text{ m}}}$ 0,5

6a



Ansatz:

$y = a(x+u)^2 + v$

$S(-u/v)$

Aus dem Graphen:

$S(4/6)$ 0,5

$P_1(8/-10)$; $P_2(0/-10)$

$\rightarrow$ S: $y = a(x-4)^2 + 6$ 0,5

P_1: $-10 = a(8-4)^2 + 6$ 0,5

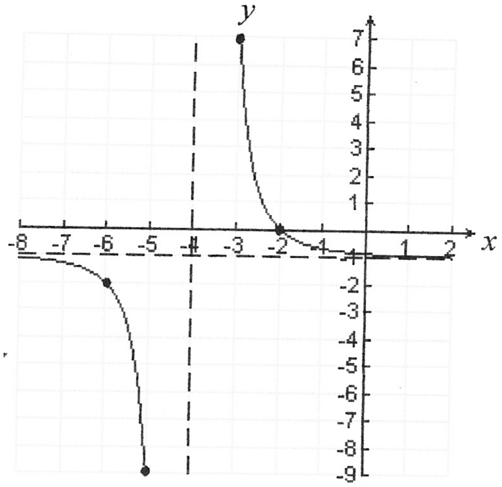
$a = \frac{-16}{16} = -1$ 0,5

$y = -(x-4)^2 + 6$ 0,5

$y = -(x^2 - 8x + 16) + 6$

$y = -x^2 + 8x - 10$

66



Ansatz:

$$y = \frac{a}{(x+b)^n} + c$$ 0,5

Aus dem Graphen:

$b = +4$ 0,5

$x \to \pm\infty \Rightarrow y \to -1$

$\Rightarrow c = -1$ 0,5

0,5 $P_1(-6|-2)$, $P_2(-5|-9)$, $P_3(-3|7)$, $P_4(-2|0)$

$$\Rightarrow y = \frac{a}{(x+4)^n} - 1$$

P_3: $7 = \frac{a}{(-3+4)^n} - 1 \Rightarrow a = 8$ 0,5+1

P_4: $0 = \frac{8}{(-2+4)^n} - 1 \Rightarrow 1 \cdot 2^n = 8$

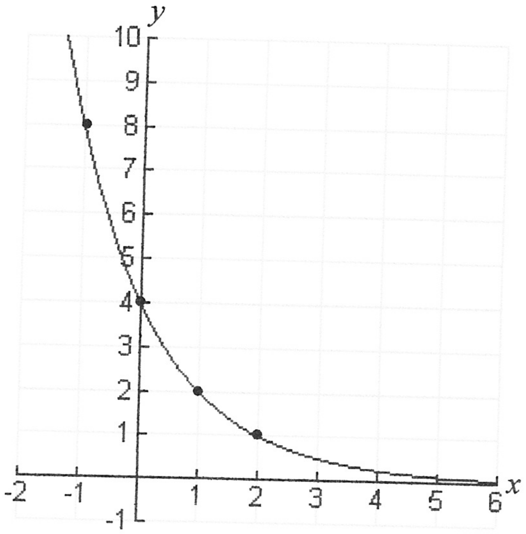
$n = 3$ 0,5+1

$$y = \frac{8}{(x+4)^3} - 1$$ 0,5+1

Kontrolle:

P_1: $-2 = \frac{8}{(-6+4)^3} - 1 \Rightarrow -2 = -2$

6c



$P_1(-1/8)$

$P_2(0/4)$

$P_3(1/2)$

$P_4(2/1)$

0,5

Ansatz

$y = f(x) = k \cdot a^{\frac{x}{d}}$ 0,5

Aus dem Graphen:

$a = \frac{1}{2}$ 0,5 (Abnahmefaktor) $\Rightarrow d = 1$ 0,5

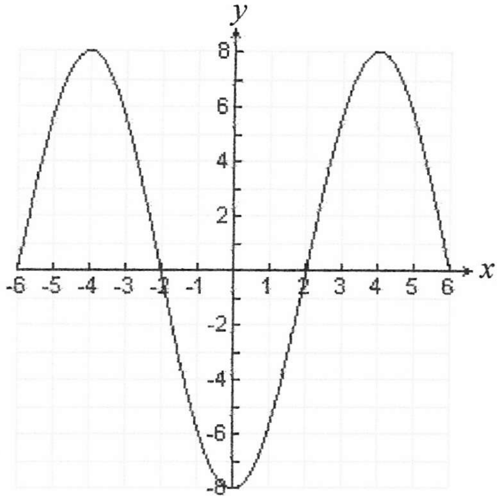
P_2: $4 = k \cdot a^0 \Rightarrow \underline{k = 4}$ 0,5

$\Rightarrow$ $\underline{\underline{y = f(x) = 4 \cdot \left(\frac{1}{2}\right)^x = 4 \cdot 2^{-x}}}$ 0,5

Kontrolle:

P_1: $8 = 4 \cdot 2^{(-1)(-1)} \Rightarrow 8$ ok.

6d



Ansätze

$y = a \sin(bx + c)$

$y = a \cdot \cos(bx)$ 0,5P

Aus dem Graphen

$|a| = 8$ 0,5P → Spiegelung an der x-Achse → (-1) (0,5P)

$p = 8$; $p_0 = 2\pi$ $\Rightarrow$ $b = \frac{p_0}{p} = \frac{2\pi}{8} = \frac{\pi}{4}$

1P

$\Rightarrow$ $y = -8 \cdot \cos\left(\frac{\pi}{4} x\right)$ 0,5P

$\Rightarrow$ $y = 8 \cdot \sin\left[\frac{\pi}{4}(x-2)\right]$ (0,5P)

↑ aus dem Graphen

$y = 8 \sin\left(\frac{\pi x}{4} - \frac{\pi}{2}\right)$ 0,5P

7a $\quad f(x) = ax^3 + bx^2 + cx + d$

Pkt: $(-1;-3),\ (0;0),\ (2;-6),\ (3;9)$

$\rightarrow \quad \underline{0 = d}$ $\quad$ 1P

$-3 = a(-1)^3 + b(-1)^2 + c(-1)$

$\underline{-3 = -a + b - c}$ $\quad$ 1P

$-6 = a(+2)^3 + b(2)^2 + c\cdot(2)$

$\underline{-6 = 8a + 4b + 2c}$ $\quad$ 1P

$9 = a\cdot 3^3 + b\cdot 3^2 + c\cdot 3$

$\underline{9 = 27a + 9b + 3c}$ $\quad$ 1P

$\rightarrow$ $\underline{3\times 3}$

$$\left|\begin{array}{l} -a + b - c = -3 \\ 8a + 4b + 2c = -6 \\ 27a + 9b + 3c = 9 \end{array}\right|$$

$\underline{\underline{a = 2}}$

$\underline{\underline{b = -4}}$ $\quad$ 2P*

$\underline{\underline{c = -3}}$

$$\left\{\begin{array}{l} D = 72 \\ D_a = 144 \\ D_b = -288 \\ D_c = -216 \end{array}\right.$$

$\left(y = 2x^3 - 4x^2 - 3x\right)$

* : Lösungsgang von Hand

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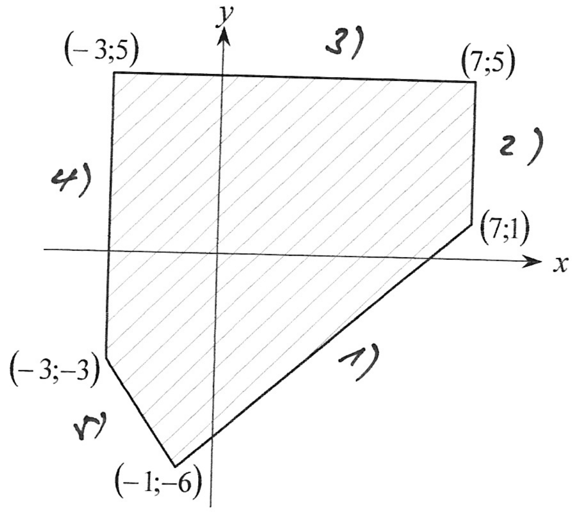

1) $y_1 = f(x) = mx + q$

$m = \frac{7}{8}$; $-6 = \frac{7}{8}(-1) + q$ 0,5

$q = -6 + \frac{7}{8} = \frac{-41}{8}$ 0,5

$y_1 \geq f(x) \geq \frac{7}{8}x - \frac{41}{8}$

2) $x \leq 7$ 0,5 3) $y \leq 5$ 0,5 4) $x \geq -3$ 0,5

5) $y_5 = f(x) = mx + q$

$m = \frac{-3}{2}$ 0,5 ; $-3 = \frac{-3}{2} \cdot (-3) + q$

$-3 - \frac{9}{2} = q \Rightarrow q = \frac{-15}{2}$ 0,5

$\Rightarrow y_5 \geq f(x) \geq \frac{-3}{2}x - \frac{15}{2}$

$$\Rightarrow \left| \begin{array}{l} y_1 \geq \frac{7x}{8} - \frac{41}{8} \\ x \leq 7 \\ y \leq 5 \\ x \geq -3 \\ y_5 \geq \frac{-3x}{2} - \frac{15}{2} \end{array} \right|$$ 0,5