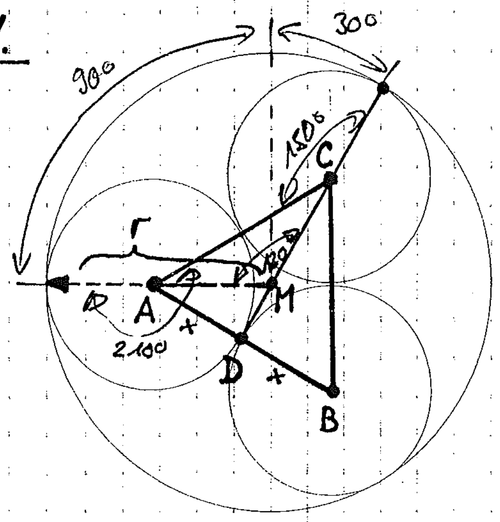
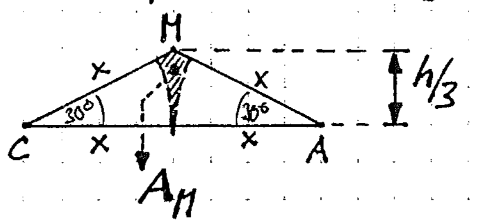
Lösungen TBM – Mathematik – Teil B

1.



$\Delta ABC \rightarrow$ gleichseitig ✓	1/4 P
$\hookrightarrow s = 2x$ ✓	1/4 P
$h \mathrel{\hat{=}} \overline{DC} = x \cdot \sqrt{3}$ ✓	1/4 P
M teilt $\overline{DC} \rightarrow 1:2$ ✓	1/4 P
Geg: $r = 10\,cm$ ✓	
$\downarrow$	
$r = \frac{2}{3} \cdot h + x$	
$r = \frac{2}{3} \cdot x\sqrt{3} + x = x\left(\frac{2}{\sqrt{3}} + 1\right)$ ✓	1/2 P
$x = \frac{10}{2/\sqrt{3} + 1} = 4{,}641\,cm$ ✓ $\Rightarrow 2x = 9{,}282\,cm$	1/2 P
Umfangslinie $= \overset{20{,}944}{\frac{2r\pi}{3}}$ ✓ $+ \overset{29{,}1604}{2x\pi}$ ✓ $= 50{,}104\,cm$ ✓	1/2 P
$A_M = \overset{12{,}4356}{\frac{x \cdot h}{3}} - \overset{11{,}2778}{\frac{x^2\pi}{6}}$ ✓	1/2 P
$A_M = \frac{x^2\sqrt{3}}{3} - \frac{x^2\pi}{6}$	
$A_M = 1{,}158\,cm^2$ ✓ (77551)	1/2 P
$A_{schraff} = \overset{104{,}719755}{\frac{r^2\pi}{3}}$ ✓ $- A_M$ ✓ $= 103{,}562\,cm^2$ ✓	1/2 P



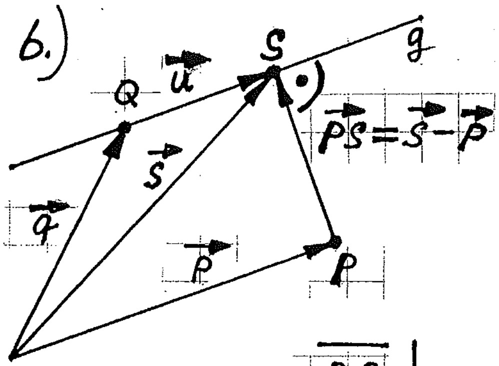
$A_{schraff} = A_{\circ} + A_{\bullet} = \underbrace{67{,}666861 + 35{,}8951387}_{103{,}5619997}$ ✓

2.

a) $\begin{pmatrix}3\\-4\\8\end{pmatrix} = \begin{pmatrix}6\\25\\8\end{pmatrix} + t\begin{pmatrix}3\\4\\0\end{pmatrix} \quad \begin{matrix}\rightarrow t=-1\\ \rightarrow t=-29/4\\ \rightarrow \underline{}\end{matrix} \Bigg\} \quad \underline{P \notin g}$ 1/2 P

$\begin{pmatrix}-3\\13\\8\end{pmatrix} = \begin{pmatrix}6\\25\\8\end{pmatrix} + t\begin{pmatrix}3\\4\\0\end{pmatrix} \quad \begin{matrix}\rightarrow t=-3\\ \rightarrow t=-3\\ \rightarrow \underline{}\end{matrix} \Bigg\} \quad \underline{Q \in g}$ 1/2 P

b)



$\overrightarrow{PS} = \begin{pmatrix}6\\25\\8\end{pmatrix} + t\begin{pmatrix}3\\4\\0\end{pmatrix} - \begin{pmatrix}3\\-4\\8\end{pmatrix}$

$\underline{\overrightarrow{PS} = \begin{pmatrix}3+3t\\29+4t\\0\end{pmatrix}}$ 1/2 P

$\overline{PS} \perp g \Rightarrow \underline{\overrightarrow{PS} \cdot \vec{u} = 0}$ (Skalarprodukt) 1/4 P

$\begin{pmatrix}3+3t\\29+4t\\0\end{pmatrix} \cdot \begin{pmatrix}3\\4\\0\end{pmatrix} = 9+9t+116+16t = 0$ 1/2 P

$125 + 25t = 0 \Rightarrow \underline{t = -5}$ 1/4 P

Mit $t=-5$ gilt:

$\vec{s} = \begin{pmatrix}6\\25\\8\end{pmatrix} + (-5)\begin{pmatrix}3\\4\\0\end{pmatrix} = \begin{pmatrix}-9\\5\\8\end{pmatrix} \Rightarrow \underline{S = (-9|5|8)}$ 1/2 P

$\underline{\overrightarrow{PS}} = \begin{pmatrix}-9\\5\\8\end{pmatrix} - \begin{pmatrix}3\\-4\\8\end{pmatrix} = \underline{\begin{pmatrix}-12\\9\\0\end{pmatrix}}$ 1/2 P

$\underline{\underline{\overline{PS}}} = \sqrt{144+81} = \underline{\underline{15}}$ 1/2 P

3.

$$\left.\begin{array}{l}\text{Mann: 57 J.}\\ \text{Frau: 59 J.}\end{array}\right\} W_0 = 50'000.- \left\{\begin{array}{l}\to 65\text{ J.} \to \underline{t_1 = 8}\\ \to 62\text{ J.} \to \underline{t_2 = 3}\end{array}\right\}$$ 1/4 P 1/4 P

beide gleich viel, bei Zinssatz Mann 4,5% u. Zinssatz Frau 3,5%. 1/2 P

$W(t_1) = (W_0 - x)\, a^{t_1} \Rightarrow W(8) = (50'000 - x)\, 1{,}045^8$ 1/2 P

$W(t_2) = x \cdot a^{t_2} \Rightarrow W(3) = x \cdot 1{,}035^3$ 1/2 P

$W(8) = W(3)$

$(50'000 - x)\, 1{,}045^8 = x \cdot 1{,}035^3$ 1/2 P

$50'000 = \frac{x \cdot 1{,}035^3}{1{,}045^8} + x = x\left(\frac{1{,}035^3}{1{,}045^8} + 1\right)$ 1/2 P

$\underline{\underline{x}} = \frac{50'000}{\frac{1{,}035^3}{1{,}045^8} + 1} = \underline{\underline{28'095{,}67\,\text{CHF}}} \to \underline{\text{Frau}}$ 1/2 P

$\underline{\underline{W_0 - x}} = 50'000 - 28'095{,}67 = \underline{\underline{21'904{,}33\,\text{CHF}}}$ 1/2 P

$\to$ <u>Mann</u>

4. $y = f(x) = 3x^3 - 4x^2$

↳ verschieben entlang $g: y = 0{,}5x + 7$ bis $P(8|11)$ vong ein Punkt der verschobenen $f(x)$ ist. $\rightarrow g(x) = ?$

- Schnittpunkt vong mit $f(x)$:

$0{,}5x + 7 = 3x^3 - 4x^2$ — 1/2 P

$3x^3 - 4x^2 - 0{,}5x - 7 = 0 \rightarrow$ Polynom

Taschenrechner (polyroot) $\rightarrow \underline{x = 2}$

$\underline{y} = f(2) = 3 \cdot 2^3 - 4 \cdot 2^2 = \underline{8}$

Schnittpunkt $S(2/8)$ — 1/2 P

- Verschiebevektor $\underline{\vec{v}} = \begin{pmatrix} 8-2 \\ 11-8 \end{pmatrix} = \underline{\begin{pmatrix} 6 \\ 3 \end{pmatrix}}$

D.h. $f(x)$ wird um 6e in x-Richtung und um 3e in y-Richtung verschoben. — 1/2 P, 1/2 P

- $g(x) = f(x-6) + 3$

$g(x) = 3(x-6)^3 - 4(x-6)^2 + 3$ — 1 P

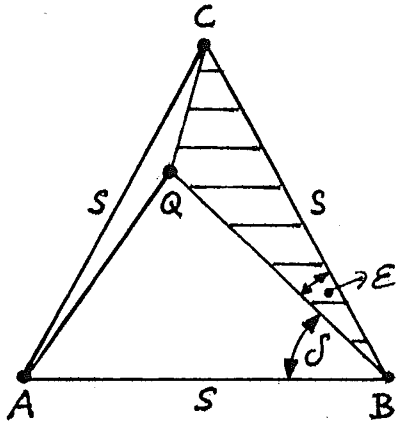
oder TR

$g(x) = 3(x^2 - 12x + 36)(x-6) - 4(x^2 - 12x + 36) + 3$

$g(x) = 3x^3 - 18x^2 - 36x^2 + 216x + 108x - 648 - 4x^2 + 48x - 144 + 3$

$$\underline{\underline{g(x) = 3x^3 - 58x^2 + 372x - 789}}$$ — 1P

5.



Geg: $s = 9\,cm$

$\overline{AQ} = 2/3 \cdot s = 6\,cm$ 1/4 P

$\overline{BQ} = 1{,}25 \cdot 2/3 \cdot s = 7{,}5\,cm$ 1/4 P

a) Kosinussatz:

$\overline{AQ}^2 = s^2 + \overline{BQ}^2 - 2 \cdot s \cdot \overline{BQ} \cdot \cos\delta$ 1/2 P

$$\cos\delta = \frac{\overline{AQ}^2 - s^2 - \overline{BQ}^2}{-2 \cdot s \cdot \overline{BQ}} = \frac{6^2 - 9^2 - 7{,}5^2}{-2 \cdot 9 \cdot 7{,}5} = 0{,}75$$ 1/2 P

$\underline{\delta = 41{,}4096°} \Rightarrow \underline{\varepsilon} = 60° - \delta = \underline{18{,}5904°}$ je 1/4 P

$$\overline{QC} = \sqrt{s^2 + \overline{BQ}^2 - 2 \cdot s \cdot \overline{BQ} \cdot \cos\varepsilon}$$

$$\underline{\underline{\overline{QC}}} = \sqrt{9^2 + 7{,}5^2 - 2 \cdot 9 \cdot 7{,}5 \cdot \cos(18{,}5904)} = \underline{\underline{3{,}0486}}\ cm$$ 1 P

b) $A_{\Delta BCQ} = \frac{1}{2} \cdot \overline{BQ} \cdot s \cdot \sin\varepsilon$

$\underline{\underline{A_{\Delta BCQ}}} = 0{,}5 \cdot 7{,}5 \cdot 9 \cdot \sin 18{,}5904° = \underline{\underline{10{,}7595\,cm^2}}$ 1 P

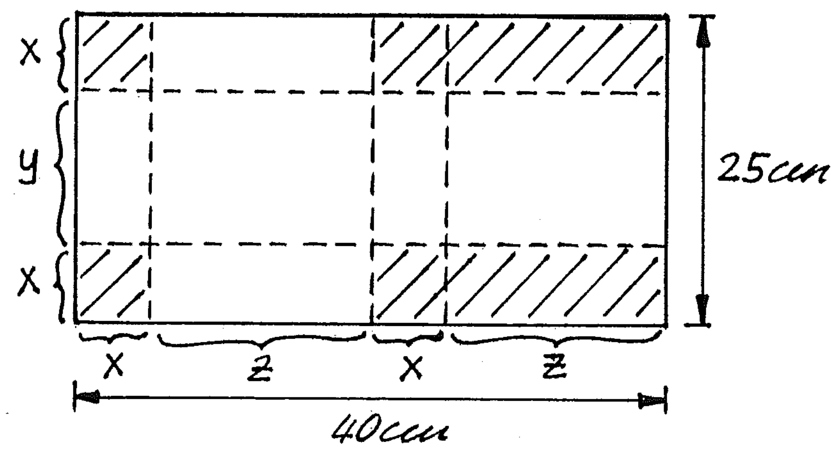
$A_{\Delta BCQ}$ kann auch mit der heronschen Formel berechnet werden:

$$\frac{u}{2} = \frac{1}{2}(\overline{QB} + s + \overline{QC}) =$$

$$\underline{\frac{u}{2}} = 0{,}5(7{,}5 + 9 + 3{,}0486) = \underline{9{,}7743}$$

$$A_{\Delta BCQ} = \sqrt{\frac{u}{2}\left(\frac{u}{2} - \overline{BQ}\right)\left(\frac{u}{2} - s\right)\left(\frac{u}{2} - \overline{QC}\right)} =$$

6.



$V_{Schachtel} = x \cdot y \cdot z \longrightarrow$ Max! ; (Zielfunktion) | 1/4 P

$40 = 2x + 2z \longrightarrow \underline{z = 20 - x}$ | 1/2 P

$25 = 2x + y \longrightarrow \underline{y = 25 - 2x}$ | 1/2 P

$V(x) = x(25-2x)(20-x)$ | 1/2 P

$V(x) = (25x - 2x^2)(20-x) = 2x^3 - 65x^2 + 500x$ | 1/4 P

CAS

mit $\mathbb{D}_V =]\,0;\,12{,}5\,[$

$V'(x) = 6x^2 - 130x + 500$

$V'(x) = 0 \Rightarrow 6x^2 - 130x + 500 = 0$

$x_{E1/2} = \dfrac{130 \pm \sqrt{130^2 - 24 \cdot 500}}{12}$ $\nearrow$ $x_{E1} = 16\,^2/_3\,cm$; $\searrow$ $\underline{\underline{x_{E2} = 5\,cm}}$

$V''(x) = 12x - 130$

$V''(x_{E2}) < 0 \Rightarrow \underline{\text{Maximum}}!$

Die Kantenlängen betragen:

$\underline{\underline{x = 5\,cm}}$ | 1 P

$\underline{\underline{y}} = 25 - 2x = \underline{\underline{15\,cm}}$ | 1/2 P

$\underline{\underline{z}} = 20 - x = \underline{\underline{15\,cm}}$ | 1/2 P