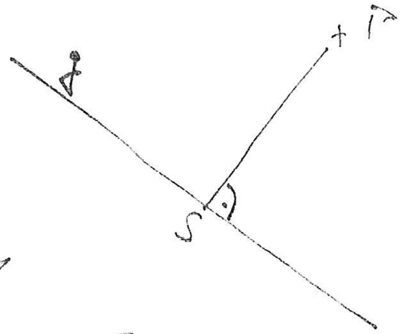
1 $g: y = f(x) = -0{,}5x + 5$ $\quad P(2/8)$



$h: y = f(x) = m_2 x + q$

$m_1 \cdot m_2 = -1 \Rightarrow -0{,}5 \cdot m_2 = -1$

$\underline{m_2 = 2}$ $\quad 0{,}5$

$h: y = f(x) = 2x + q$

$P: 8 = 4 + q \rightarrow \underline{q = 4}$ $\quad 0{,}5$

$h: y = f(x) = 2x + 4$ $\quad 0{,}5$

$\Rightarrow -0{,}5x + 5 = 2x + 4$ $\quad 0{,}5$

$2{,}5x = 1$

$\underline{x = 0{,}4}$ $\quad 0{,}5$

$\underline{y} = -0{,}5 \cdot 0{,}4 + 5 = \underline{4{,}8}$ $\quad 0{,}5$

$\underline{\underline{S(0{,}4/4{,}8)}}$

2

$$y = f(x) = ax^3 + bx + c$$

$P_1 \, (-3/-30)$

$P_2 \, (0/0) \quad \rightarrow \quad f(0) = 0 \Rightarrow \underline{\underline{c = 0}}$ 0,5

$P_3 \, (1/-6)$

$$\begin{array}{l} P_1: \\ P_3: \end{array} \left| \begin{array}{l} -30 = -27a - 3b \\ -6 = a + b \end{array} \right|$$ 1P

$\rightarrow \quad b = -6 - a$

$\rightarrow \quad 30 = 27a + 3(-6 - a)$

$30 = 27a - 18 - 3a$

$48 = 24a$

$\underline{\underline{a = 2}} \rightarrow \underline{\underline{b = -8}}$ 1,5 P

$\underline{\underline{y = f(x) = 2x^3 - 8x}}$

3

$$\frac{x+4}{|x-5|} > 2 \qquad \underline{\underline{x \neq 5}}$$

$$x + 4 > 2|x-5| \quad 0{,}5$$

<u>1. Fall</u>

$$x-5 > 0$$

$$\underline{x > 5} \quad 0{,}5$$

$$\Rightarrow \quad x+4 > 2x-10$$

$$\underline{14 > x} \quad 0{,}5$$

$$\Rightarrow \underline{\mathbb{L}_1 =]5;14[}$$

0,5

<u>2. Fall</u>

$$\underline{x < 5} \quad 0{,}5$$

$$x+4 > -2(x-5)$$

$$x+4 > -2x+10$$

$$3x > 6$$

$$\underline{x > 2} \quad 0{,}5$$

$$\underline{\mathbb{L}_2 =]2;5[}$$

0,5

$$\Rightarrow \mathbb{L} = \mathbb{L}_1 \cup \mathbb{L}_2$$

$$\underline{\underline{\mathbb{L} =]2;14[\setminus \{5\}}} \quad 0{,}5$$

4.a) $a\cdot e^{0,1x} = b \qquad a, b \in \mathbb{R}^+$

$$e^{0,1x} = \frac{b}{a} \quad 0,5 \quad | \ln(\,)$$

$$0,1x = \ln\left(\frac{b}{a}\right) \quad 1P$$

$$\underline{\underline{x = 10\cdot\ln\left(\frac{b}{a}\right)}} \quad 0,5$$

4b)

$$\underset{①}{\frac{1}{\ln(x)}} + \frac{1}{2} = \overset{②}{\frac{\ln(x)}{2}}$$

$\underline{D}$: ① $\ln(x) \Rightarrow x > 0$ 0,5P

$\ln(x) \neq 0 \Rightarrow x \neq 1$ 0,5P

② $x > 0$

$$\Rightarrow \underline{\underline{D = \,]0;\infty[\, \setminus \{1\}}} \quad 0,5P$$

$$\frac{1}{\ln(x)} + \frac{1}{2} = \frac{\ln(x)}{2} \qquad | 2\cdot\ln(x)$$

$$2 + \ln(x) = \ln(x)^2 \quad 0,5P$$

$$\ln(x)^2 - \ln(x) - 2 = 0 \quad 0,5P$$

$\underline{\text{Substitution}}$ $\quad u = \ln(x)$

$$\Rightarrow u^2 - u - 2 = 0$$

$$(u+1)(u-2) = 0 \quad 0,5P$$

$u_1 = -1 \qquad \underline{\underline{x_1}} = e^{-1} = \underline{\underline{\frac{1}{e}}}$

$u_2 = 2 \qquad \underline{\underline{x_2 = e^2}} \qquad 1P$

5.

$$\vartheta(t) = \vartheta_u + (\vartheta_0 - \vartheta_u)e^{-\lambda t}$$

$$\vartheta_u = 20°C$$

$$\vartheta_0 = 100°C$$

$$\vartheta(10) = 30°C$$

a)

$$\Rightarrow \quad 30 = 20 + (100-20)e^{-10\lambda} \qquad 0{,}5\ P$$

$$10 = 80\,e^{-10\lambda}$$

$$\frac{1}{8} = e^{-10\lambda} \qquad | \ln$$

$$-10\lambda = \ln\left(\frac{1}{8}\right)$$

$$\underline{\underline{\lambda}} = \frac{\ln\left(\frac{1}{8}\right)}{-10} = \underline{\underline{0{,}208}} \qquad 2{,}5\ P$$

b) 1. Art

$$\vartheta_{01} = \frac{100+4}{2} = 52°C \qquad 1\ P$$

$$t = 5\ \text{min}$$

$$\vartheta_1(5) = 20 + (52-20)e^{-0{,}208\cdot 5}$$

$$\underline{\underline{\vartheta_1(5) = 31{,}31°C}} \qquad 1{,}5\ P$$

2. Art

$$\vartheta_{02} = \vartheta_{21} = 20 + (100-20)\cdot e^{-0{,}208\cdot 5}$$

$$\vartheta_{02} = 48{,}28°C \qquad 1{,}5\ P$$

$$\underline{\underline{\vartheta_2}} = \frac{\vartheta_{02}+4}{2} = \frac{48{,}28+4}{2} = \underline{\underline{26{,}14°C}} \qquad 1\ P$$

6a)

$$\frac{1}{\sin(2x)} - \cot(2x) = \frac{1}{\sin(2x)} - \frac{\cos(2x)}{\sin(2x)} \quad 0{,}5$$

0,5

$$= \frac{1-\cos(2x)}{\sin(2x)} = \frac{1-(1-2\sin^2(x))}{2\sin(x)\cos(x)} \quad 0{,}5 \quad 0{,}5$$

$$= \frac{\cancel{2}\sin^{\cancel{2}}(x)}{\cancel{2}\cancel{\sin(x)}\cos(x)} \; 0{,}5 \; = \frac{\sin(x)}{\cos(x)} = \underline{\underline{\tan(x)}} \quad 0{,}5$$

6b)

$$2-2\cos^2(x) = 3\sin(2x) \qquad \mathbb{D} = [0;360°[$$

$$2-2\cos^2(x) = 3\cdot 2\sin(x)\cos(x) \quad 0{,}5$$

$$2(1-\cos^2(x)) = 6\sin(x)\cos(x) \qquad 2P$$

$$\cancel{2}\sin^2(x) = \cancel{6}^{3}\sin(x)\cos(x)$$

$$\sin^{\cancel{2}}(x) = 3\cancel{\sin(x)}\cos(x) \qquad /:\sin(x)$$

$$\sin(x) = 3\cdot\cos(x) \qquad \sin(x) = 0 \quad 1P$$

$$\frac{\sin(x)}{\cos(x)} = 3 \qquad x = 0°, 180°$$

(Lösungen)

$$\underline{\tan(x) = 3}$$

$$\underline{x} = \arctan(3) = \underline{71{,}6°} \quad 0{,}5$$

Periodizität: $x = 180° + 71{,}6°$

$$\underline{x = 251{,}6°} \quad 0{,}5P$$

$$\Rightarrow \underline{\underline{\mathbb{L} = \{0°; 71{,}6°; 180°; 251{,}6°\}}} \quad 0{,}5$$

7a $\quad \vec{v} = \begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} a-1 \\ -2 \\ 1 \end{pmatrix}, \quad A = 3$

$$A = |\vec{v} \times \vec{w}|$$

$$\Rightarrow \quad \vec{v} \times \vec{w} = \begin{pmatrix} a \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} a-1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} 0 & -2 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} a & a-1 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} a & a-1 \\ 0 & -2 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -(a-a+1) \\ -2a \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -2a \end{pmatrix}$$

$$A = \sqrt{2^2 + (-1)^2 + (-2a)^2}$$

$$A^2 = 4 + 1 + 4a^2$$

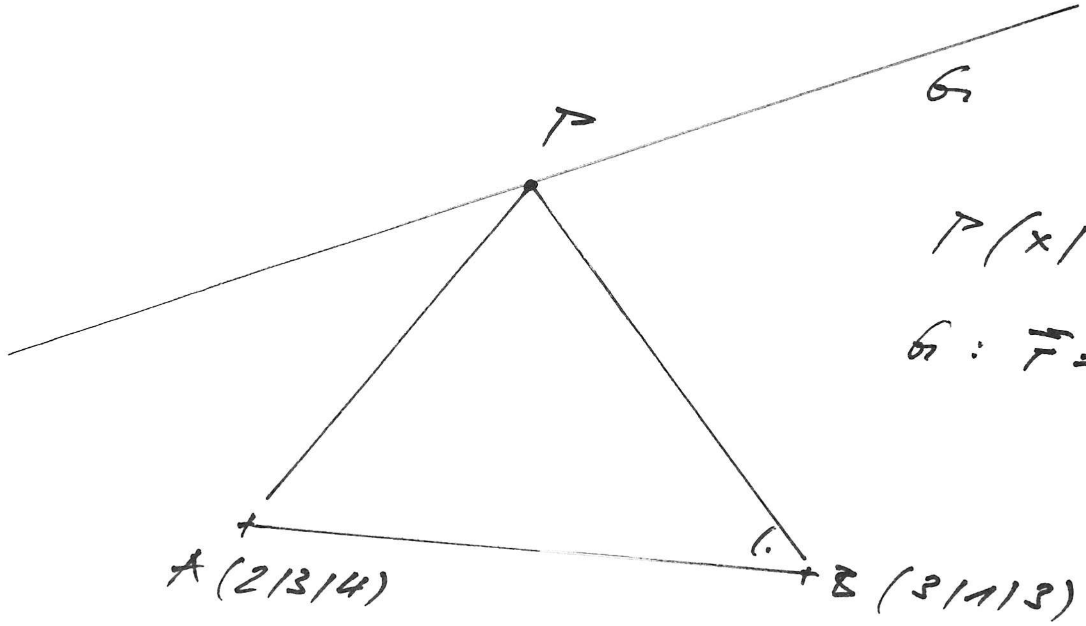
$$\Rightarrow \quad 9 = 5 + 4a^2$$

$$a^2 = 1$$

$$|a| = 1$$

$$\underline{\underline{a = \pm 1}}$$

76



$P(x|y|z)$

$g: \vec{r} = \begin{pmatrix} -\lambda \\ 2\lambda \\ 3\lambda \end{pmatrix}$

$\vec{AB} \cdot \vec{BP} = 0$

$\vec{AB} = \vec{r}_B - \vec{r}_A = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$

$\vec{AB} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ 0,5

$\vec{BP} = \vec{r}_P - \vec{r}_B$

$= \begin{pmatrix} -\lambda \\ 2\lambda \\ 3\lambda \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$

$\vec{r}_P = \begin{pmatrix} -\lambda \\ 2\lambda \\ 3\lambda \end{pmatrix}$ 0,5

$\vec{BP} = \begin{pmatrix} -\lambda - 3 \\ 2\lambda - 1 \\ 3\lambda - 3 \end{pmatrix}$ 0,5

$\rightarrow \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \begin{pmatrix} -\lambda - 3 \\ 2\lambda - 1 \\ 3\lambda - 3 \end{pmatrix} = 0$ 0,5

$-\lambda - 3 - 2(2\lambda - 1) - 1(3\lambda - 3) = 0$

$-\lambda - \cancel{3} - 4\lambda + 2 - 3\lambda + \cancel{3} = 0$

$8\lambda = 2$

$\lambda = 1/4$

0,5 P

$\vec{r}_P = \begin{pmatrix} -1/4 \\ 1/2 \\ 3/4 \end{pmatrix}$ 0,5 $\Leftarrow$

7c) $P: \vec{r} = \vec{r}_A + \lambda(\vec{r}_B - \vec{r}_A)$

$$\vec{r} = \begin{pmatrix} 6 \\ 4 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3-6 \\ 0-4 \\ 4-5 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ -4 \\ -1 \end{pmatrix}$$

$$P: \vec{r} = \begin{pmatrix} 6 \\ 4 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$$ 1P

$$S: \vec{r} = \vec{r}_P + \mu \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -7 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$ 0,5P

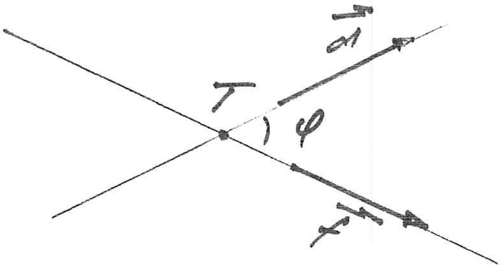
Schnittpunkt T:

$$\left| \begin{array}{l} 6 + 3\lambda = 0 \\ 4 + 4\lambda = -7 + \mu \\ 5 + \lambda = 0 + \mu \end{array} \right| \Rightarrow \left| \begin{array}{l} 3\lambda = -6 \\ 4\lambda - \mu = -11 \\ \lambda - \mu = -5 \end{array} \right|$$

2×2

$$\left| \begin{array}{l} 4\lambda - \mu = -11 \\ \lambda - \mu = -5 \end{array} \right| \quad \lambda = -2, \quad \mu = 3$$ 1P

$$\Rightarrow T: \vec{r}_T = \begin{pmatrix} 0 \\ -7 \\ 0 \end{pmatrix} + 3 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix}$$ 0,5P

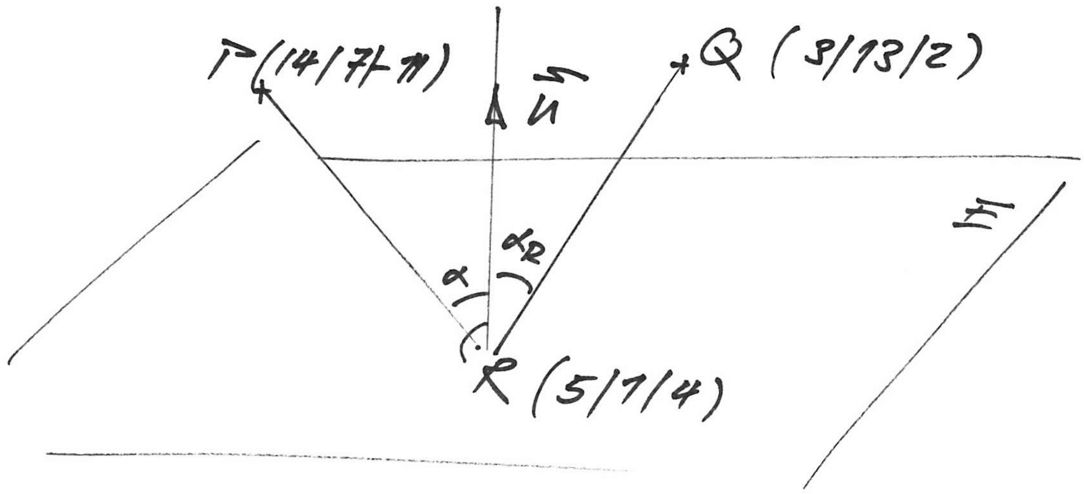


$$\cos\varphi = \frac{\vec{d} \cdot \vec{f}}{|\vec{d}| \cdot |\vec{f}|} = \frac{\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}}{\sqrt{26} \cdot \sqrt{2}}$$

$$\cos\varphi = \frac{5}{\sqrt{26} \cdot \sqrt{2}} = 0{,}693$$ 1P

$$\varphi = 46{,}1^\circ$$

7d



$$\vec{RP} = \vec{r}_P - \vec{r}_R = \begin{pmatrix}14\\7\\-11\end{pmatrix} - \begin{pmatrix}5\\1\\4\end{pmatrix} = \begin{pmatrix}9\\6\\-15\end{pmatrix}$$ 0,5

$$\vec{RQ} = \vec{r}_Q - \vec{r}_R = \begin{pmatrix}3\\13\\2\end{pmatrix} - \begin{pmatrix}5\\1\\4\end{pmatrix} = \begin{pmatrix}-2\\12\\-2\end{pmatrix}$$ 0,5

$$\vec{u} = \frac{\vec{RP}}{|\vec{RP}|} + \frac{\vec{RQ}}{|\vec{RQ}|}$$ 0,5

$$|\vec{RP}| = \sqrt{81+36+225}$$

$$\underline{|\vec{RP}| = 3\sqrt{38}}$$ 0,5

$$|\vec{RQ}| = \sqrt{4+144+4}$$

$$|\vec{RQ}| = 2\sqrt{38}$$ 0,5

$$\vec{u} = \frac{\begin{pmatrix}9\\6\\-15\end{pmatrix}}{3\sqrt{38}} + \frac{\begin{pmatrix}-2\\12\\-2\end{pmatrix}}{2\sqrt{38}}$$

$$\sqrt{38}\cdot\vec{u} = \underline{\vec{u}\,'} = \begin{pmatrix}3\\2\\-5\end{pmatrix} + \begin{pmatrix}-1\\6\\-1\end{pmatrix} = \underline{\begin{pmatrix}2\\8\\-6\end{pmatrix}}$$ 0,5

$$E:\ \vec{u}\,'(\vec{r} - \vec{r}_R) = 0$$

$$\begin{pmatrix}2\\8\\-6\end{pmatrix}\left[\begin{pmatrix}x\\y\\z\end{pmatrix} - \begin{pmatrix}5\\1\\4\end{pmatrix}\right] = 0$$ 0,5

$$2x + 8y - 6z - (10 + 8 - 24) = 0$$ 0,5

$$2x + 8y - 6z + 6 = 0 \Rightarrow \underline{\underline{x + 4y - 3z + 3 = 0}}$$

__8.__ Anzahl Mathematikbücher : x

Anzahl Physikbücher : y

0,5P

$x, y \in \mathbb{N}$, $x, y \geq 0$

__Bedingungen__

1) $x \geq 50$ 0,5P

2) $y \geq 80$ 0,5

3) $x + y \geq 200 \Rightarrow y \geq -x + 200$ 1P

4) $32x + 40y \leq 9000 \Rightarrow y = \frac{-4x}{5} + 225$ 1P

5) $z = 5x + 6y$ (Min) $\rightarrow y = \frac{-5x}{6} + \frac{z}{6}$ 1P

Lösungen : $\underline{\underline{x = 120}}$, $\underline{\underline{y = 80}}$ 0,5P

Wertminderung:

$z = 120 \cdot 5 + 6 \cdot 80$

$\underline{\underline{z = 1080 \text{ CHF}}}$ 0,5P

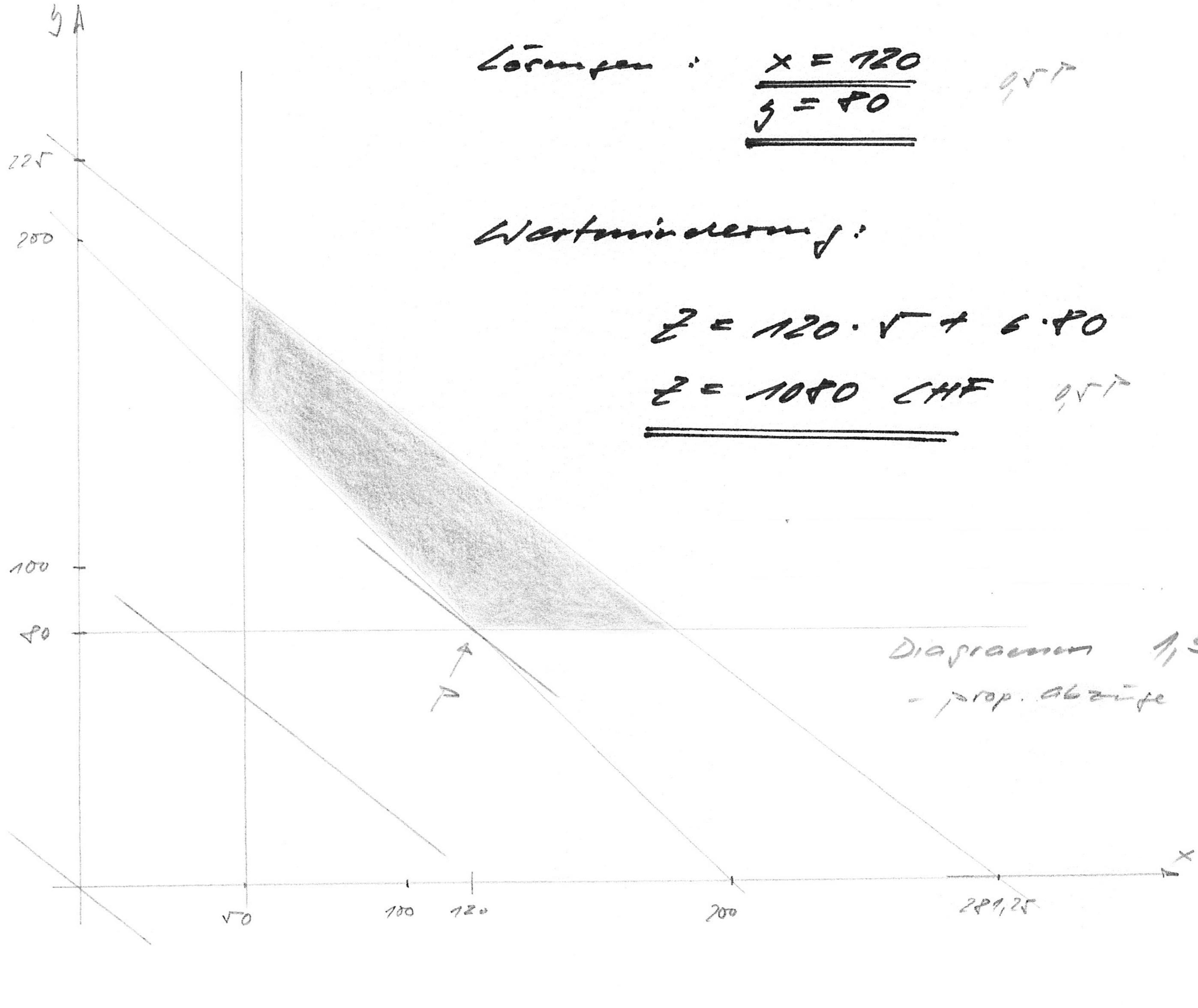
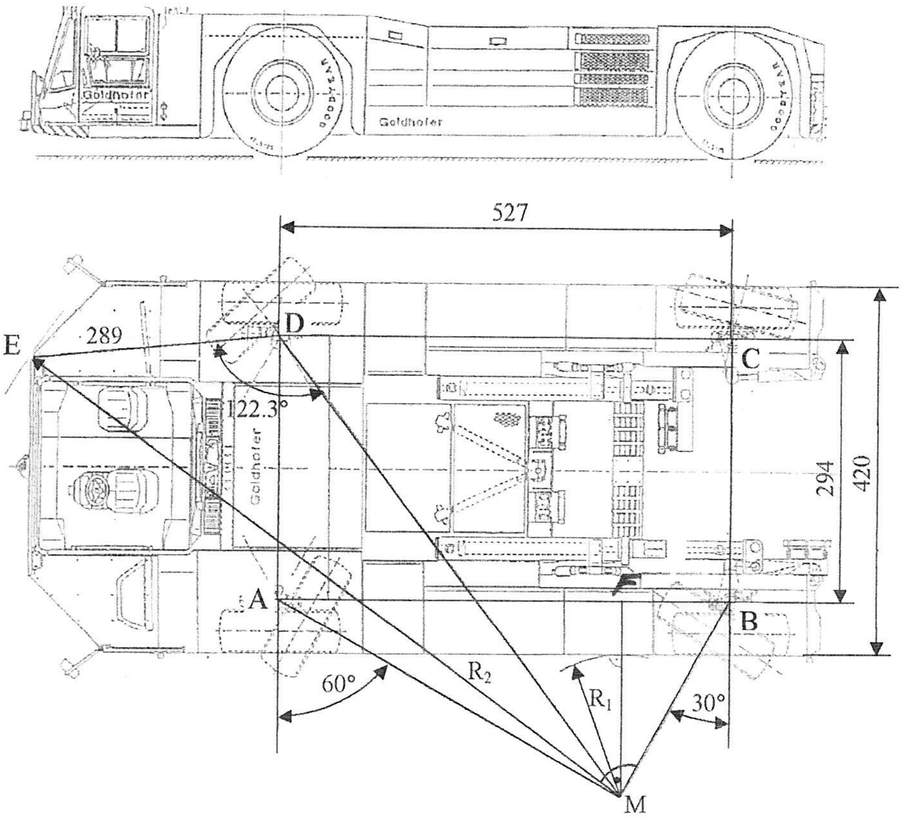


Diagramm 1,5

- prop. Abzüge



a) $\overline{MB} = \dfrac{\overline{AB}}{2} = \dfrac{527}{2} = 263{,}5\ \text{cm}$ 0,5P

$\overline{MF} = \dfrac{\overline{MB}}{2}\sqrt{3} = \dfrac{263{,}5}{2}\sqrt{3} = 228{,}2\ \text{cm}$ 0,5P

$\underline{\underline{R_1}} = \overline{MF} - \dfrac{420-294}{2} = \underline{\underline{165\ \text{cm}}}$ 1P

b) $\overline{AF} = \overline{MF}\cdot\sqrt{3} = 228{,}2\cdot\sqrt{3} = 395{,}3\ \text{cm}$ 0,5P

$\overline{MD}^2 = \overline{AF}^2 + (\overline{MF} + \overline{AD})^2$

$\underline{MD} = \sqrt{395{,}3^2 + (228{,}2 + 294)^2} = \underline{654{,}9\ \text{cm}}$ 1P

$R_2^2 = \overline{DE}^2 + \overline{MD}^2 - 2\,\overline{DE}\,\overline{MD}\cos 122{,}3^\circ$

$R_2^2 = \sqrt{289^2 + 654{,}9^2 - 2\cdot 289\cdot 654{,}9\cos 122{,}3^\circ}$

$\underline{\underline{R_2 = 845\ \text{cm}}}$ 1,5P