Aufgabe 1

$$3 - \frac{x-5}{x+2} \geq \frac{2x}{x-1}$$

$$3 - \frac{x-5}{x+2} - \frac{2x}{x-1} \geq 0$$

$$\frac{3(x+2)(x-1) - (x-5)(x-1) - 2x(x+2)}{(x+2)(x-1)} \geq 0$$

$$\frac{3x^2+3x-6-x^2+6x-5-2x^2-4x}{(x+2)(x-1)} \geq 0$$

$$\frac{5x-11}{(x+2)(x-1)} \geq 0$$

I) $5x - 11 \geq 0 \longrightarrow 5x \geq 11 \longrightarrow x \geq \frac{11}{5}$

$(x+2)\cdot(x-1) > 0$

a) $x+2 > 0 \quad \wedge \quad x-1 > 0$

$x > -2 \qquad x > 1$

$\longrightarrow L_{Ia} = \{x \in \mathbb{R} \mid x \geq \frac{11}{5}\}$

b) $x+2 < 0 \quad \wedge \quad x-1 < 0$

$x < -2 \qquad x < 1$

$\longrightarrow L_{Ib} = \{\}$

II) $5x - 11 \leq 0 \longrightarrow 5x \leq 11 \longrightarrow x \leq \frac{11}{5}$

$(x+2)(x-1) < 0$

a) $x+2 > 0 \quad \wedge \quad x-1 < 0$

$x > -2 \qquad x < 1$

$\longrightarrow L_{IIa} = \{x \in \mathbb{R} \mid -2 < x < 1\}$

b) $x+2 < 0 \quad \wedge \quad x-1 > 0$

$x < -2 \qquad x > 1$

$\longrightarrow L_{IIb} = \{\}$

$$L = \{x \in \mathbb{R} \mid x \geq \frac{11}{5} \quad \vee \quad -2 < x < 1\}$$

Aufgabe 2

$$2 \cdot \sqrt{4-x} - \sqrt{3(x+4)} = \sqrt{3x+44}$$

$$4 \cdot (4-x) - 2 \cdot 2 \cdot \sqrt{(4-x) \cdot 3 \cdot (x+4)} + 3(x+4) = 3x + 44$$

$$16 - 4x - 4\sqrt{48 - 3x^2} + 3x + 12 = 3x + 44$$

$$-4\sqrt{48-3x^2} = 4x + 16$$

$$-\sqrt{48-3x^2} = x + 4$$

$$48 - 3x^2 = x^2 + 8x + 16$$

$$0 = 4x^2 + 8x - 32$$

$$0 = x^2 + 2x - 8$$

$$x_{1/2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-8)}}{2 \cdot 1} = \frac{-2 \pm \sqrt{36}}{2} = \frac{-2 \pm 6}{2}$$

$$x_1 = \frac{4}{2} = 2$$

$$x_2 = \frac{-8}{2} = -4$$

Kontrolle:

$x_1 = 2$:

$$2 \cdot \sqrt{2} - \sqrt{3 \cdot 6} = \sqrt{6 + 44}$$

$$2\sqrt{2} - \sqrt{18} = \sqrt{50}$$

$$2\sqrt{2} - 3 \cdot \sqrt{2} = 5 \cdot \sqrt{2}$$ ✗

$x_2 = -4$:

$$2 \cdot \sqrt{8} - \sqrt{3 \cdot 0} = \sqrt{-12 + 44}$$

$$4 \cdot \sqrt{2} - 0 = \sqrt{32}$$

$$4 \cdot \sqrt{2} = 4 \cdot \sqrt{2}$$ ✓

$$\underline{\underline{L = \{-4\}}}$$

Aufgabe 3

	Arbeitszeit alleine (Tage)	Arbeit pro Tag
Gärtner	x	$\frac{1}{x}$
Lehrling	y	$\frac{1}{y}$

(1) $12 \cdot 7 \cdot \frac{1}{x} + 12 \cdot 2 \cdot \frac{1}{y} = 1$

(2) $5 \cdot 6 \cdot \frac{1}{x} + 10 \cdot 3 \cdot \frac{1}{x} + 10 \cdot 3 \cdot \frac{1}{y} = 1$

(1): $\frac{84}{x} + \frac{24}{y} = 1$

(2): $\frac{80}{x} + \frac{30}{y} = 1$

$5 \cdot$(1): $\frac{420}{x} + \frac{120}{y} = 5$

$-4 \cdot$(2): $-\frac{320}{x} - \frac{120}{y} = -4$

$\frac{100}{x} = 1$

$x = 100$

in (1): $\frac{80}{100} + \frac{30}{y} = 1$

$\frac{30}{y} = \frac{1}{5}$

$y = 150$

Aufgabe 4

$$\log_5(7x) = 4 - \log_3 x^2$$

$$\frac{\lg 7x}{\lg 5} = 4 - \frac{\lg x^2}{\lg 3}$$

$$\lg 7x \cdot \lg 3 = 4 \cdot \lg 5 \cdot \lg 3 - \lg x^2 \cdot \lg 5$$

$$(\lg 7 + \lg x) \cdot \lg 3 = 4 \cdot \lg 5 \cdot \lg 3 - 2 \cdot \lg x \cdot \lg 5$$

$$\lg 7 \cdot \lg 3 + \lg x \cdot \lg 3 = 4 \lg 5 \cdot \lg 3 - 2 \lg x \cdot \lg 5$$

$$\lg x \cdot \lg 3 + 2 \cdot \lg x \cdot \lg 5 = 4 \cdot \lg 5 \cdot \lg 3 - \lg 7 \cdot \lg 3$$

$$\lg x \,(\lg 3 + 2 \cdot \lg 5) = 4 \cdot \lg 5 \cdot \lg 3 - \lg 7 \cdot \lg 3$$

$$\lg x = \frac{4 \lg 5 \cdot \lg 3 - \lg 7 \cdot \lg 3}{\lg 3 + 2 \lg 5}$$

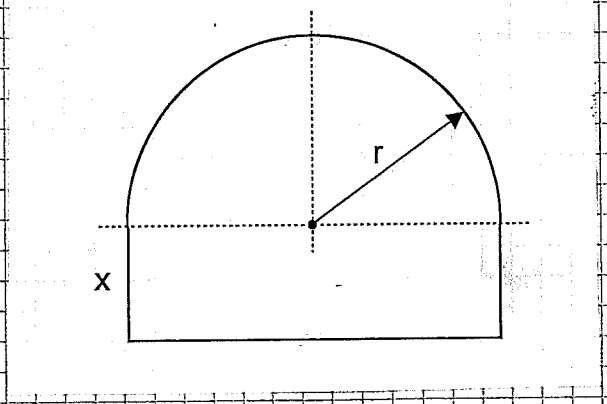
$$= 0{,}4964$$

$$x = 10^{0{,}4964}$$

$$x = 3{,}1361$$

$$\underline{\underline{L = \{3{,}14\}}}$$

Aufgabe 5



$$r \cdot \pi + 2r + 2x = 100$$

$$2x = 100 - r\pi - 2r$$

$$x = \frac{100 - r\pi - 2r}{2}$$

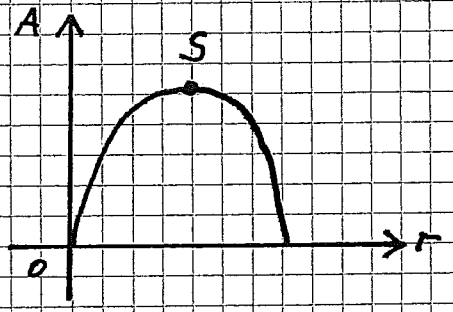
$$A = \frac{r^2\pi}{2} + 2r \cdot x$$

$$= \frac{r^2\pi}{2} + 2r \cdot \left(\frac{100 - r\pi - 2r}{2}\right)$$

$$= \frac{r^2\pi}{2} + r(100 - r\pi - 2r)$$

$$= \frac{r^2\pi}{2} + 100r - r^2\pi - 2r^2$$

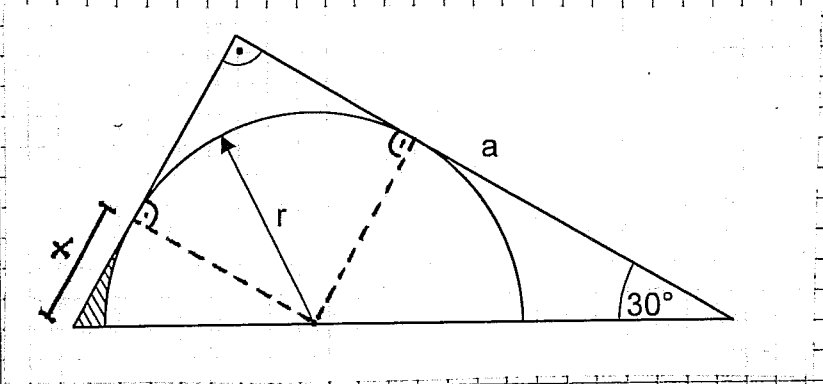
$$= r^2 \cdot \frac{-\pi - 4}{2} + 100 \cdot r \longrightarrow \max$$



$$x_s = -\frac{b}{2a} = -\frac{100}{2 \cdot \frac{-\pi - 4}{2}} = \frac{100}{\pi + 4} = 14,0025$$

$$\underline{\underline{r = 14\,cm}}$$

Aufgabe 6



$$a = r + r \cdot \sqrt{3}$$

$$a = r(1 + \sqrt{3})$$

$$r = \frac{a}{1+\sqrt{3}} = a \cdot 0{,}366$$

$$r = x \cdot \sqrt{3}$$

$$x = \frac{r}{\sqrt{3}} = \frac{a}{\sqrt{3}(1+\sqrt{3})}$$

$$A = \frac{r \cdot x}{2} - \frac{r^2 \pi}{12}$$

$$= \frac{a \cdot a}{(1+\sqrt{3}) \cdot \sqrt{3} \cdot (1+\sqrt{3}) \cdot 2} - \frac{a^2 \pi}{12 \cdot (1+\sqrt{3})^2}$$

$$= a^2 \frac{2 \cdot \sqrt{3} - \pi}{(1+\sqrt{3})^2 \cdot 12}$$

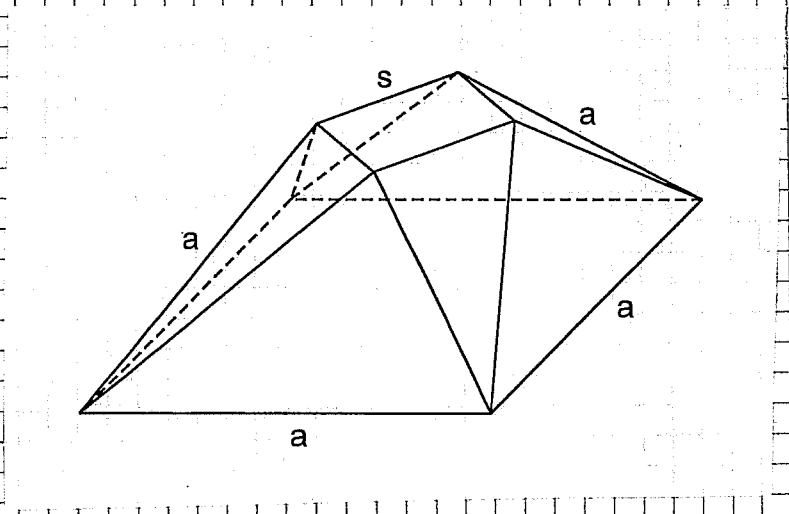
$$= a^2 \frac{2 \cdot \sqrt{3} - \pi}{12 \cdot (4 + 2\sqrt{3})} = a^2 \frac{(2\sqrt{3} - \pi)(4 - 2\sqrt{3})}{12(4+2\sqrt{3})(4-2\sqrt{3})}$$

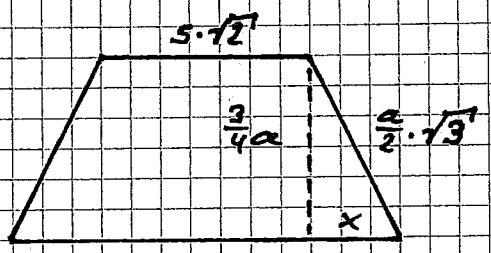
$$= a^2 \frac{(2\sqrt{3} - \pi)(4 - 2\sqrt{3})}{12 \cdot (16 - 12)} = a^2 \frac{(2 \cdot \sqrt{3} - \pi) \cdot 2(2 - \sqrt{3})}{12 \cdot 4}$$

$$= a^2 \frac{(2\sqrt{3} - \pi)(2 - \sqrt{3})}{24}$$

$$= a^2 \cdot 0{,}0036$$

Aufgabe 7





$$x = \sqrt{\left(\frac{a}{2} \cdot \sqrt{3}\right)^2 - \left(\frac{3}{4}a\right)^2} = \sqrt{\frac{3a^2}{4} - \frac{9a^2}{16}}$$

$$= \sqrt{\frac{3a^2}{16}} = \frac{a\sqrt{3}}{4}$$

$$a = 2x + s \cdot \sqrt{2}$$

$$a = 2 \cdot \frac{a \cdot \sqrt{3}}{4} + s \cdot \sqrt{2}$$

$$a = \frac{a \cdot \sqrt{3}}{2} + s \cdot \sqrt{2}$$

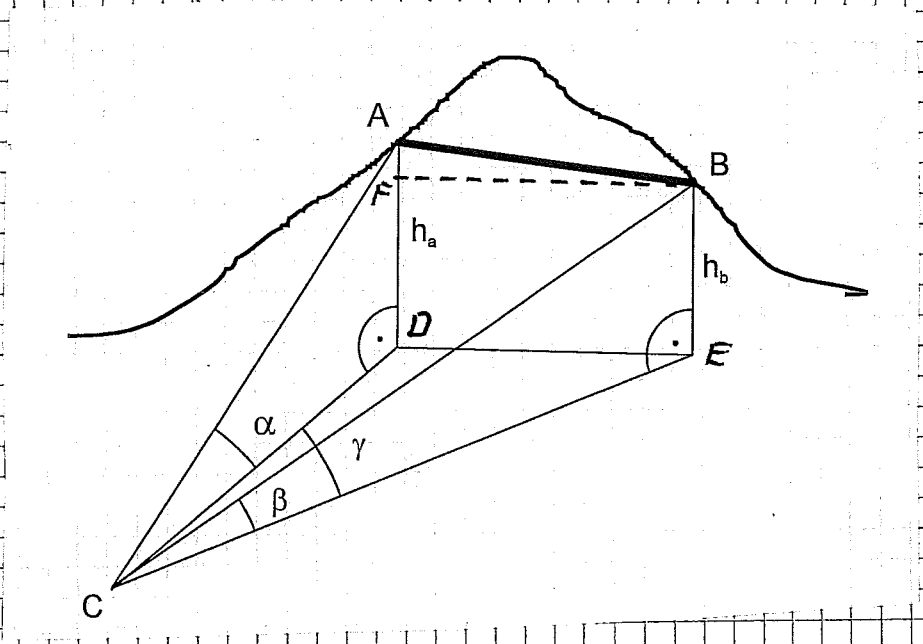
$$s \cdot \sqrt{2} = a - \frac{a \cdot \sqrt{3}}{2}$$

$$s \cdot \sqrt{2} = a \cdot \frac{2 - \sqrt{3}}{2}$$

$$s = a \cdot \frac{2 - \sqrt{3}}{2 \cdot \sqrt{2}}$$

$$s = a \cdot 0{,}0947$$

Aufgabe 8



$$\tan\alpha = \frac{h_a}{\overline{CD}} \rightarrow \overline{CD} = \frac{h_a}{\tan\alpha} = \frac{61,3}{\tan 10,8°} = \underline{321,3459\,m}$$

$$\tan\beta = \frac{h_b}{\overline{CE}} \rightarrow \overline{CE} = \frac{h_b}{\tan\beta} = \frac{52,3}{\tan 8,4°} = \underline{354,1749\,m}$$

$$\overline{DE} = \sqrt{\overline{CD}^2 + \overline{CE}^2 - 2\cdot\overline{CD}\cdot\overline{CE}\cdot\cos\gamma}$$

$$= \sqrt{321,35^2 + 354,17^2 - 2\cdot 321,35\cdot 354,17\cdot\cos 53,3°}$$

$$= 304,4149\,m$$

$$\overline{AB} = \sqrt{\overline{AF}^2 + \overline{FB}^2}$$

$$= \sqrt{(61,3 - 52,3)^2 + 304,41^2}$$

$$= \sqrt{9^2 + 304,41^2}$$

$$= \underline{\underline{304,5479\,m}}$$

Aufgabe 9

$$\frac{1}{2} = \cos^2\left(\frac{\alpha}{2}\right) - \sin(2\alpha)$$

$$\frac{1}{2} = \frac{\cos\alpha + 1}{2} - 2\sin\alpha\cos\alpha$$

$$\frac{1}{2} = \frac{\cos\alpha}{2} + \frac{1}{2} - 2\sin\alpha\cos\alpha$$

$$0 = \frac{\cos\alpha}{2} - 2\sin\alpha\cos\alpha$$

$$0 = \cos\alpha - 4\sin\alpha\cos\alpha$$

$$0 = \cos\alpha\,(1 - 4\sin\alpha)$$

I) $\cos\alpha = 0 \longrightarrow \alpha_1 = 90°$

$\alpha_2 = 270°$

II) $1 - 4\sin\alpha = 0$

$4\sin\alpha = 1$

$\sin\alpha = \frac{1}{4} \longrightarrow \alpha_3 = 14,48°$

$\alpha_4 = 165,52°$

$$\underline{\underline{L = \{14,58°;\ 90°;\ 165,52°;\ 270°\}}}$$

Aufgabe 10

a) $P(0/y/0)$

$$\overline{AP} = 3\,\overline{BP}$$

$$\sqrt{(0+5)^2+(y-7)^2+(0-10)^2} = 3\cdot\sqrt{(0-3)^2+(y-1)^2+(0-0)^2}$$

$$25+y^2-14y+49+100 = 9(9+y^2-2y+1)$$

$$y^2-14y+174 = 9y^2-18y+90$$

$$0 = 8y^2-4y-84$$

$$0 = 2y^2-y-21$$

$$y_{1/2} = \frac{-(-1)\pm\sqrt{(-1)^2-4\cdot2\cdot(-21)}}{2\cdot2} = \frac{1\pm13}{4}$$

$$y_1 = \frac{14}{4} = 3{,}5 \quad / \quad y_2 = \frac{-12}{4} = -3$$

$$\underline{\underline{P_1(0/3{,}5/0)}} \qquad \underline{\underline{P_2(0/-3/0)}}$$

b) I)

$$\cos\alpha_1 = \frac{\overrightarrow{P_1A}\cdot\overrightarrow{P_1B}}{\overline{P_1A}\cdot\overline{P_1B}} = \frac{\begin{pmatrix}-5-0\\7-3{,}5\\10-0\end{pmatrix}\cdot\begin{pmatrix}3-0\\1-3{,}5\\0-0\end{pmatrix}}{\sqrt{(-5-0)^2+(7-3{,}5)^2+(10-0)^2}\cdot\sqrt{(3-0)^2+(1-3{,}5)^2+0^2}}$$

$$= \frac{\begin{pmatrix}-5\\3{,}5\\10\end{pmatrix}\begin{pmatrix}3\\-2{,}5\\0\end{pmatrix}}{\sqrt{5^2+3{,}5^2+10^2}\cdot\sqrt{3^2+2{,}5^2}} = \frac{-15-8{,}75}{11{,}72\cdot3{,}91} = \frac{-23{,}75}{45{,}75}$$

$$= -0{,}5191 \longrightarrow \underline{\underline{\alpha_1 = 121{,}27^\circ}}$$

II)

$$\cos\alpha_2 = \frac{\overrightarrow{P_2A}\cdot\overrightarrow{P_2B}}{\overline{P_2A}\cdot\overline{P_2B}} = \frac{\begin{pmatrix}-5-0\\7-(-3)\\10-0\end{pmatrix}\cdot\begin{pmatrix}3-0\\1-(-3)\\0-0\end{pmatrix}}{\sqrt{(-5-0)^2+(7-(-3))^2+(10-0)^2}\cdot\sqrt{(3-0)^2+(1-(-3))^2+0^2}}$$

$$= \frac{\begin{pmatrix}-5\\10\\10\end{pmatrix}\begin{pmatrix}3\\4\\0\end{pmatrix}}{\sqrt{5^2+10^2+10^2}\cdot\sqrt{3^2+4^2}} = \frac{-15+40}{15\cdot5} = \frac{25}{75}$$

$$= 0{,}\overline{3} \longrightarrow \underline{\underline{\alpha_2 = 70{,}53^\circ}}$$