Schulinterne Sperrfrist bis Ende Juni 2010

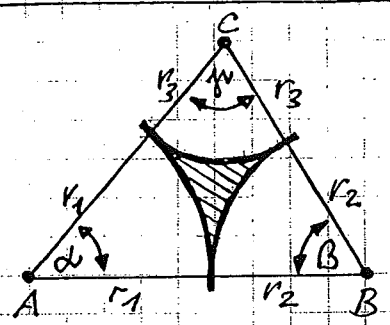
TECHNISCHE BERUFSMATURITÄT

Mathematik Teil B

Lösungen

Matura prüfung 2009
Lösungen Mathematik Teil B

1.



$r_1 = 5\,cm$; $r_2 = 4\,cm$; $r_3 = 3\,cm$

$\Rightarrow a = r_2 + r_3 = 7\,cm$

$b = r_3 + r_1 = 8\,cm$

$c = r_1 + r_2 = 9\,cm$ 0,5P

$\cos\alpha = \frac{7^2 - 8^2 - 9^2}{-2 \cdot 8 \cdot 9} = \frac{2}{3} \Rightarrow \underline{\alpha = 48{,}19°}$ 0,5P

$\cos\beta = \frac{8^2 - 9^2 - 7^2}{-2 \cdot 9 \cdot 7} = \frac{11}{21} \Rightarrow \underline{\beta = 58{,}41°}$ 0,5P

$\gamma = 180° - 48{,}19° - 58{,}41° = \underline{73{,}40°}$ 0,5P

$A_Y = A_\Delta - A_\alpha - A_\beta - A_\gamma$ ↗

$= \frac{8 \cdot 9 \cdot \sin\alpha}{2} - \frac{5^2 \cdot \pi \cdot \alpha}{360} - \frac{4^2 \cdot \pi \cdot \beta}{360} - \frac{3^2 \cdot \pi \cdot \gamma}{360}$ 0,5P

$= 26{,}8328 - 10{,}5134 - 8{,}1558 - 5{,}7647 = \underline{2{,}399}\ cm^2$

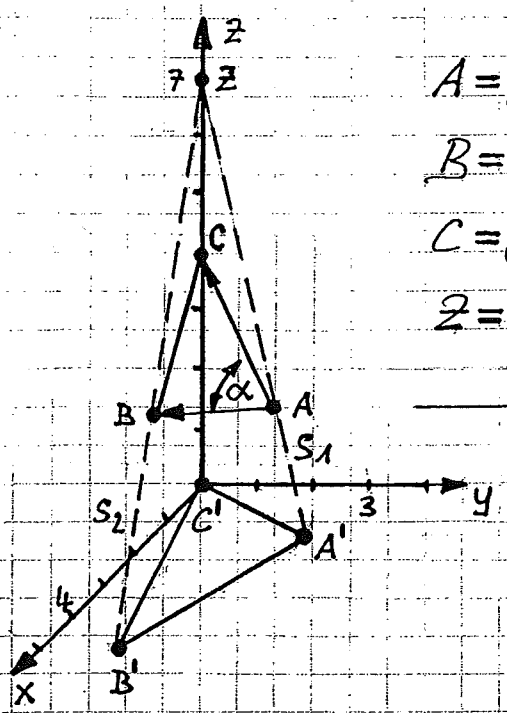
$\Rightarrow$ 0,5P

$U_Y = U_\alpha + U_\beta + U_\gamma$ ↗

$= \frac{10 \cdot \pi \cdot \alpha}{360} + \frac{8 \cdot \pi \cdot \beta}{360} + \frac{6 \cdot \pi \cdot \gamma}{360}$ 0,5P

$= 4{,}2053 + 4{,}0779 + 3{,}8431 = \underline{12{,}126\,cm}$ 0,5P

2.



$A=(1/2/2)$

$B=(3/1/3)$

$C=(0/0/4)$

$Z=(0/0/7)$

a.) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ 0,25P

$\overrightarrow{AC} = \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}$ 0,25P

$\cos\alpha = \frac{2\cdot(-1)+(-1)(-2)+1\cdot 2}{\sqrt{6}\cdot\sqrt{9}}$ 0,25P

$\alpha = 74,2068°$ 0,25P

b.) $C' = (0/0/0)$ 0,25P

$S_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$ 0,25P

$A': \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix} \rightarrow t = 7/5$ 0,25P

$x = 0 + 7/5 = 7/5$

$y = 0 + 2\cdot 7/5 = 14/5$ } $A' = (1,4/2,8/0)$ 0,5P

$S_2: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix}$ 0,25P

$B': \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \\ -4 \end{pmatrix} \rightarrow t = 7/4$ 0,25P

$x = 0 + 3\cdot 7/4 = 21/4$

$y = 0 + 7/4 = 7/4$ } $B' = (5,25/1,75/0)$ 0,5P

c.) $\overline{A'B'} = \sqrt{(5,25-1,4)^2 + (1,75-2,8)^2} = 3,99$ 0,75P

3. a.) $A(t) = A_0 \cdot a^t$ mit $A_0 = \frac{2 \cdot 10^6}{8} m^2$ 0,5P

a. $a = 1,15$ folgt: $A(t) = 2,5 \cdot 10^5 \cdot 1,15^t$ 0,5P

b.) $A(4) = 2,5 \cdot 10^5 \cdot 1,15^4 = 437251,5625 m^2$ 1P

c.) $2A_0 = A_0 \cdot 1,15^t$ 2 0,5P

$t = \frac{\lg 2}{\lg 1,15} = 4,96$ Tagen 0,5P

d.) $2 \cdot 10^6 = 2,5 \cdot 10^5 \cdot 1,15^t$ 2 0,5P

$t = \frac{\lg 8}{\lg 1,15} = 14,878$ Tagen 0,5P

4. a.) $g = f_1(x) = (x+4)(x+1)^2(x-4)$ 0,5P

$= (x^2-16)(x^2+2x+1)$

$= x^4+2x^3+x^2-16x^2-32x-16$ 0,5P

$g = f_1(x) = \frac{1}{16}x^4 + \frac{1}{8}x^3 - \frac{15}{16}x^2 - 2x - 1$ 0,5P

$g = f_2(x) = -(x-2)(x+3)(x+6)$ 0,5P

$= -(x^2+x-6)(x+6)$

$= -x^3-7x^2+36$ 0,5P

$g = f_2(x) = -\frac{1}{12}x^3 - \frac{7}{12}x^2 + 3$ 0,5P

b.) $g = f_1(x) = (x+3)x^2(x-5) \cdot \frac{1}{16}$ 0,5P

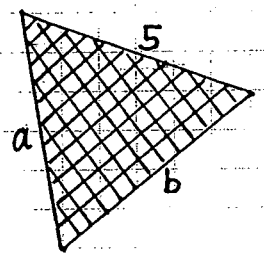
$= (x^3+3x^2)(x-5) \cdot \frac{1}{16}$

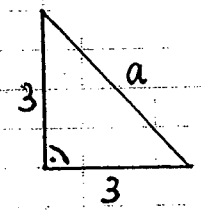
$= (x^4-2x^3-15x^2) \cdot \frac{1}{16}$

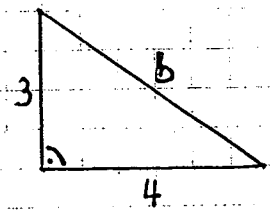
$g = f_1(x) = \frac{1}{16}x^4 - \frac{1}{8}x^3 - \frac{15}{16}x^2$ 0,5P

5.

a.)







$a = 3\sqrt{2}$ 0,5P

$b = 5$ 0,5P

$s = \frac{a+b+5}{2} = 7{,}12132$ 0,5P

$A_{III} = \sqrt{s(s-a)(s-b)(s-5)} = 9{,}605\,cm^2$ 0,5P

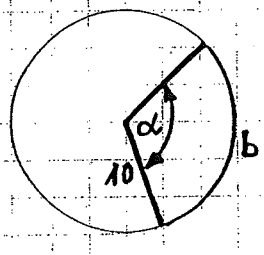
b.) $V_{Körper} = V_{Prisma} - V_{Pyramide}$ 0,5P

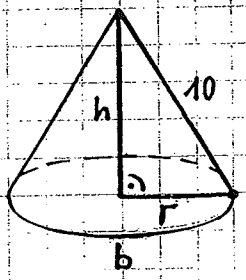
$V_{Prisma} = \sqrt{6(6-3)(6-4)(6-5)} \cdot 9 = 54\,cm^3$ 0,5P

$V_{Pyramide} = \sqrt{6(6-3)(6-4)(6-5)} \cdot \frac{3}{3} = 6\,cm^3$ 0,5P

$V_{Körper} = 54\,cm^3 - 6\,cm^3 = 48\,cm^3$ 0,5P

6.





$b = \frac{2 \cdot \pi \cdot 10 \cdot \alpha}{360} = \frac{\pi \cdot \alpha}{18}$ 0,5P

$b = 2 \cdot \pi \cdot r$

$r = \frac{b}{2 \cdot \pi} = \frac{\alpha}{36}$ 1P

$h = \sqrt{100 - r^2} = \sqrt{100 - \left(\frac{\alpha}{36}\right)^2}$ 0,5P

$V_{Kegel} = \frac{\pi}{3}\left(\frac{\alpha}{36}\right)^2 \cdot \sqrt{100 - \left(\frac{\alpha}{36}\right)^2}$ 1P

Rechnen: V_{Kegel} → Maximum für $\alpha = 293{,}94°$ 1P

$V_{Kegel} = \frac{\pi}{3}\left(\frac{100 \cdot \alpha^4}{36^4} - \frac{\alpha^6}{36^6}\right)^{1/2}$

$V'_{Kegel} = \frac{\pi}{6}\left(\frac{100 \cdot \alpha^4}{36^4} - \frac{\alpha^6}{36^6}\right)^{-1/2} \cdot \left(\frac{400 \cdot \alpha^3}{36^4} - \frac{6 \cdot \alpha^5}{36^6}\right)$

$V'_{Kegel} = 0 \Rightarrow 400\alpha^3 - \frac{6 \cdot \alpha^5}{36^2} = 0 \Rightarrow \alpha_{1/2/3} = 0$

$\alpha_{4/5} = \pm\sqrt{\frac{400 \cdot 36^2}{6}} = \pm 120 \cdot \sqrt{6} = \pm 293{,}94°$