BMP 09
Mathe 1

1) a) geg: A(0/3/6); B(1/-4/5); C(7/9/-7); D(4/6/-4)

ges: $\sphericalangle \vec{a}\vec{b}$, mit $\vec{a} = \overrightarrow{AB}$, $\vec{b} = \overrightarrow{CD}$

$$\cos\varphi = \frac{\vec{a} \circ \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{15}{\sqrt{51}\sqrt{27}} = 0{,}404$$

$\Rightarrow \underline{\underline{\varphi = 66{,}2°}}$

b) geg: $\vec{u}$, $\vec{r}$, $\vec{s}$, $\vec{A}$

ges: Linearkombination

$$\vec{u} = x\vec{r} + y\vec{s} + z \cdot \vec{A}$$

$$\begin{pmatrix} -11 \\ -12 \\ 1 \end{pmatrix} = x\begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix} + y\begin{pmatrix} -3 \\ 5 \\ 5 \end{pmatrix} + z\begin{pmatrix} -8 \\ -6 \\ 4 \end{pmatrix}$$

$$\left| \begin{array}{l} -11 = 2x - 3y - 8z \\ -12 = 5x + 5y - 6z \\ 1 = -2x + 5y + 4z \end{array} \right|$$

$\Rightarrow x = 1, \quad y = -1, \quad z = 2$

$\Rightarrow \underline{\underline{\vec{u} = \vec{r} - \vec{s} + 2\vec{A}}}$ 2

2) geg: $\begin{pmatrix}1\\y\\0\end{pmatrix}, \begin{pmatrix}0\\y\\1\end{pmatrix}, \varphi = 60°$

ges: y

$$\begin{pmatrix}1\\y\\0\end{pmatrix} \circ \begin{pmatrix}0\\y\\1\end{pmatrix} = \left\|\begin{pmatrix}1\\y\\0\end{pmatrix}\right\| \cdot \left\|\begin{pmatrix}0\\y\\1\end{pmatrix}\right\| \cdot \cos 60°$$

$$y^2 = \sqrt{1+y^2} \cdot \sqrt{y^2+1} \cdot \frac{1}{2}$$

$$2y^2 = \left(\sqrt{1+y^2}\right)^2$$

$$2y^2 = 1+y^2$$

$$y^2 = 1 \Rightarrow \underline{\underline{y_1 = +1}} \quad \underline{\underline{y_2 = -1}}$$ 2

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3) geg: A, B, C

ges: a) U des Δ' ABC

b) A

a) $U = |\vec{AB}| + |\vec{BC}| + |\vec{CA}| = \underline{\underline{13,87}}$

b) $\cos \measuredangle = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| \cdot |\vec{AC}|} = \frac{-4}{\sqrt{6}\sqrt{26}} = -0,3206$

$\Rightarrow \measuredangle = 108,68°$

$A = \frac{1}{2} |\vec{AB}| \cdot |\vec{AC}| \cdot \sin \measuredangle = \underline{\underline{5,92}}$ 2

Variante: $A = \frac{1}{2} \sqrt{|\vec{AB}|^2 \cdot |\vec{AC}|^2 - (\vec{AB} \circ \vec{AC})^2}$

4) geg: Gerade $\vec{r}$

ges: a) Spurpunkte S_1, S_2

b) $|\overrightarrow{S_1 S_2}|$

a) $S_1 = \begin{pmatrix} x \\ 0 \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \Rightarrow t = -\frac{1}{2}$

$\rightarrow \underline{\underline{S_1 = \begin{pmatrix} 2,5 \\ 0 \\ 3,5 \end{pmatrix}}}$

$S_2 = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \Rightarrow t = -4$

$\Longrightarrow \underline{\underline{S_2 = \begin{pmatrix} 6 \\ -7 \\ 0 \end{pmatrix}}}$

b) $|\overrightarrow{S_1 S_2}| = \left| \begin{pmatrix} +3,5 \\ -7 \\ -3,5 \end{pmatrix} \right| = \underline{\underline{8,57}}$ 2

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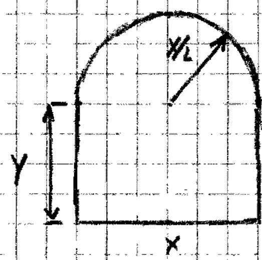
5) geg: Lüftungskanal, $U = 160\,cm$



a) $U = \frac{x}{2}\pi + 2y + x = 160$

$y = \dfrac{160 - \frac{x\pi}{2} - x}{2}$

$= 80 - \dfrac{x(\pi + 2)}{4}$

$A = \dfrac{\left(\frac{x}{2}\right)^2 \cdot \pi}{2} + xy = \dfrac{x^2\pi}{8} + x\left(80 - \dfrac{x(\pi+2)}{4}\right)$

$= \dfrac{x^2\pi}{8} + 80x - x^2\left(\dfrac{\pi+2}{4}\right) = x^2\left(\dfrac{\pi}{8} - \dfrac{\pi+2}{4}\right) + 80x$

$= \left(\dfrac{-\pi - 4}{8}\right) x^2 + 80x$

TR $A \rightarrow$ maximal $\Longrightarrow$ $\underline{\underline{x = 44,8\,cm}}$

b) $y = ax^2 + bx + c$

$= -\frac{(\pi+4)}{8}x^2 + 80x$

$\Rightarrow a = -\frac{(\pi+4)}{8}, \; b = 80, \; c = 0$

$S\left(\frac{-b}{2a} \Big/ \frac{4ac-b^2}{4a}\right) = \left(\frac{-80}{-\frac{(\pi+4)}{4}} \Big/ \frac{-80^2}{-\frac{(\pi+4)}{2}}\right)$

$= \underline{\underline{\left(\frac{320}{\pi+4} \Big/ \frac{12800}{\pi+4}\right) = (44{,}8 / 1792{,}3)}}$ 2

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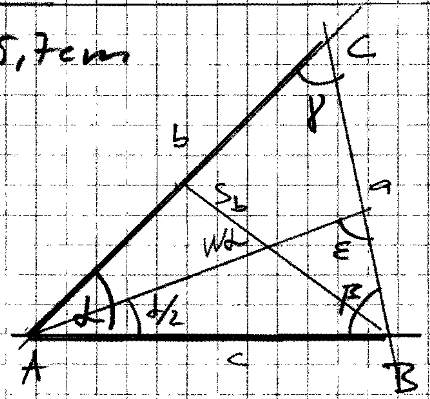
6) geg: Δ aus $b = 12{,}5\,cm$, $c = 15{,}7\,cm$

$\alpha = 58{,}7°$

ges: a, β, w_α, s_b



a) $a = \sqrt{b^2 + c^2 - 2bc\cos\alpha}$

$= \underline{\underline{13{,}68\,cm}}$

b) $\frac{\sin\beta}{b} = \frac{\sin\alpha}{a}$

$\Rightarrow \sin\beta = \frac{b \cdot \sin\alpha}{a} \Rightarrow \underline{\underline{\beta = 49{,}8°}}$

c) $\varepsilon = 180° - \frac{\alpha}{2} - \beta = 101{,}87°$

$\frac{w_\alpha}{\sin\beta} = \frac{c}{\sin\varepsilon} \Rightarrow w_\alpha = \frac{c \cdot \sin\beta}{\sin\varepsilon} = \underline{\underline{12{,}25\,cm}}$

d) $s_b = \sqrt{\left(\frac{b}{2}\right)^2 + c^2 - 2\,\frac{b}{2} \cdot c\cos\alpha}$

$= \underline{\underline{13{,}33\,cm}}$ 2

7) geg: p: $y = x^2 + bx + c$, $A(-3/1)$, $B(1/5)$
$P(2/0)$; $Q(4/4)$

ges: a) $f(x)$, S
b) Schnittpunkte S_1, S_2 mit $y = x^2 + 3x + \frac{41}{16}$

a)
$$\left| \begin{array}{l} 1 = 9 - 3b + c \\ 5 = 1 + b + c \end{array} \right|$$

$$\left| \begin{array}{l} -8 = -3b + c \\ 4 = b + c \end{array} \right| \Rightarrow c = 4 - b = 1$$

$$\left| \begin{array}{l} -8 = -3b + c \\ -4 = -b - c \end{array} \right|$$

$$-12 = -4b \Rightarrow b = 3$$

$$\underline{\underline{y = x^2 + 3x + 1}}$$

$$S\left(\frac{-b}{2a} \Big/ \frac{4ac - b^2}{4a}\right) = \underline{\underline{\left(-\frac{3}{2} \Big/ -\frac{5}{4}\right)}}$$ 2

b) $P(2/0)$, $Q(4/4)$
$\Rightarrow M(3/2)$

$$a = \frac{\Delta y}{\Delta x} = \frac{4-0}{4-2} = 2$$

$$a' = -\frac{1}{2}$$

Mittelsenkrecht

$$2 = -\frac{3}{2} + b \Rightarrow b = \frac{7}{2}, \quad y = -\frac{1}{2}x + \frac{7}{2}$$

$$\left| \begin{array}{l} y = -\frac{1}{2}x + \frac{7}{2} \\ y = x^2 + 3x + \frac{41}{16} \end{array} \right| \quad x_1 = -\frac{15}{4} \quad x_2 = \frac{1}{4} \quad \underline{\underline{S_1\left(-\frac{15}{4} \Big/ \frac{43}{8}\right); S_2\left(\frac{1}{4} \Big/ \frac{27}{8}\right)}}$$ 2

$S_1(-3{,}75/5{,}375)$; $S_2(0{,}25/3{,}375)$

8) a) $\sin(\varphi) + \sin\left(\varphi + \frac{2\pi}{3}\right) + \sin\left(\varphi + \frac{4\pi}{3}\right)$

$= \sin(\varphi) + \sin\varphi\cos\left(\frac{2\pi}{3}\right) + \cos\varphi\sin\left(\frac{2\pi}{3}\right)$

$+ \sin\varphi\cos\left(\frac{4\pi}{3}\right) + \cos\varphi\sin\left(\frac{4\pi}{3}\right)$

$= \sin(\varphi) - \frac{1}{2}\sin(\varphi) + \frac{\sqrt{3}}{2}\cos(\varphi) - \frac{1}{2}\sin(\varphi) - \frac{\sqrt{3}}{2}\cos(\varphi)$

$= \underline{\underline{0}}$

$$\frac{\sin(2t)}{1+\cos(2t)} = \frac{\sin t\cos t + \sin t\cos t}{1+\cos^2 t - \sin^2 t}$$

$$= \frac{2\sin t\cos t}{1+\cos^2 t - (1-\cos^2 t)} = \frac{2\sin t\cos t}{2\cos^2 t} = \frac{\sin t}{\cos t}$$

$$= \underline{\underline{\tan t}}$$

b) $2(1-\cos(2x)) = 3\sin(2x)$

$2 - 2\cos(2x) = 3\sin(2x) \quad |^2$

$4 - 8\cos(2x) + 4\cos^2(2x) = 9\sin^2(2x)$

$4 - 8\cos(2x) + 4\cos^2(2x) = 9(1-\cos^2(2x))$

$= 9 - 9\cos^2(2x)$

$13\cos^2(2x) - 8\cos(2x) - 5 = 0$

$13y^2 - 8y - 5 = 0$

$\Rightarrow y_1 = \frac{-5}{13}, \quad y_2 = 1$

$y_1 = \frac{-5}{13} \Rightarrow x_1 = \frac{\arccos\left(\frac{-5}{13}\right)}{2} = 0,98277$

$y_2 = 1 \Rightarrow x_2 = \frac{\arccos(1)}{2} = 0 + k\cdot\pi$

$\underline{\underline{x_1 = 0, \quad x_2 = 0,983, \quad x_3 = \pi}}$