

## Aufgabe 1

x: Alkoholgehalt der Sorte B (%)

$$14 \cdot \frac{53}{100} = 5 \cdot \frac{98}{100} + 7 \cdot \frac{x}{100} + 2 \cdot \frac{0}{100}$$

$$14 \cdot 53 = 5 \cdot 98 + 7x$$

$$742 = 490 + 7x$$

$$252 = 7x$$

$$\underline{\underline{x = 36}}$$

## Aufgabe 2

$$\frac{1}{x-m} = \frac{2}{3x-2m} + \frac{5}{3x+m}$$

$$\underline{\underline{D = \mathbb{R} \setminus \left\{ -\frac{m}{3}, \frac{2m}{3}, m \right\}}}$$

$$(3x-2m) \cdot (3x+m) = 2 \cdot (x-m)(3x+m) + 5 \cdot (x-m)(3x-2m)$$

$$9x^2 + 3xm - 6xm - 2m^2 = 6x^2 + 2xm - 6xm - 2m^2 + 15x^2 - 10xm - 15xm + 10m^2$$

$$9x^2 + 3xm - 2m^2 = 21x^2 - 29xm + 8m^2$$

$$0 = 12x^2 - 26xm + 10m^2$$

$$0 = 6x^2 - 13xm + 5m^2$$

$$x_{1/2} = \frac{-(-13m) \pm \sqrt{(-13m)^2 - 4 \cdot 6 \cdot 5m^2}}{2 \cdot 6}$$

$$= \frac{13m \pm \sqrt{49m^2}}{12} = \frac{13m \pm 7m}{12}$$

$$x_1 = \frac{13m + 7m}{12} = \frac{20m}{12} = \underline{\underline{\frac{5m}{3}}}$$

$$x_2 = \frac{13m - 7m}{12} = \frac{6m}{12} = \underline{\underline{\frac{m}{2}}}$$

$$\underline{\underline{L = \left\{ \frac{m}{2}, \frac{5m}{3} \right\}}}$$

### Aufgabe 3

x: Länge der Autokolonne (km)

y: Durchschnittliche Geschwindigkeit Kolonne (km/h)

$$s = v \cdot t$$

$$\textcircled{1} \quad x = (162 + y) \cdot \frac{1}{30}$$

$$\textcircled{2} \quad x = (162 - y) \cdot \frac{1}{24}$$

$$(162 + y) \cdot \frac{1}{30} = (162 - y) \cdot \frac{1}{24}$$

$$24 \cdot (162 + y) = 30 \cdot (162 - y)$$

$$3888 + 24y = 4860 - 30y$$

$$54y = 972$$

$$\underline{\underline{y = 18}}$$

$$x = (162 + 18) \cdot \frac{1}{30} = 180 \cdot \frac{1}{30} = \underline{\underline{6}}$$

### Aufgabe 4

$$\frac{3 \sqrt{\left(\frac{p}{p+q}\right)^{-2}} \cdot 4 \sqrt{1 + \frac{p}{q}}}{\sqrt{q} \cdot \sqrt[3]{p^{-14}} \cdot \sqrt[6]{(p+q)^5}} = \frac{\left(\frac{p}{p+q}\right)^{-\frac{3}{2}} \cdot \left(\frac{q+p}{q}\right)^{\frac{1}{4}}}{q^{\frac{1}{2}} p^{-\frac{14}{6}} \cdot (p+q)^{\frac{5}{6}}}$$

$$= p^{-\frac{2}{3}} (p+q)^{\frac{2}{3}} \cdot (p+q)^{\frac{1}{4}} q^{-\frac{1}{4}} q^{-\frac{1}{2}} p^{\frac{14}{6}} (p+q)^{-\frac{5}{6}}$$

$$= (p+q)^{\frac{8+3-10}{12}} \cdot p^{\frac{-4+14}{6}} \cdot q^{\frac{-1-2}{4}}$$

$$= \underline{\underline{(p+q)^{\frac{1}{12}} \cdot p^{\frac{5}{6}} \cdot q^{-\frac{3}{4}}}}$$

## Aufgabe 5

a)  $c = 6$

$$\rightarrow y = \log_a(x+6) + c$$

$$S: 7 = \log_a(-1+6) + c = \log_a 5 + c$$

$$T: 10 = \log_a(19+6) + c = \log_a 25 + c$$

$$c = 7 - \log_a 5 = 10 - \log_a 25$$

$$\log_a 25 - \log_a 5 = 3$$

$$\log_a \frac{25}{5} = 3$$

$$\log_a 5 = 3$$

$$\rightarrow a^3 = 5$$

$$\underline{a = 5^{\frac{1}{3}}}$$

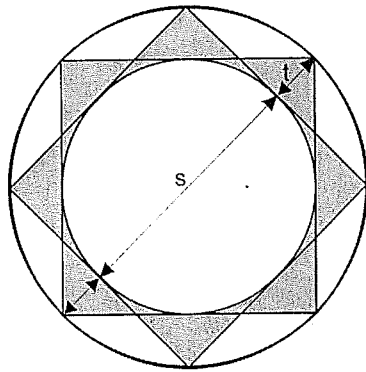
$$c = 7 - \log_{5^{\frac{1}{3}}} 5 = 7 - \frac{\log 5}{\log 5^{\frac{1}{3}}} = 7 - 3 = 4$$

$$\rightarrow \underline{\underline{y = \log_{5^{\frac{1}{3}}}(x+6) + 4}}$$

b)  $\log_{5^{\frac{1}{3}}} z = \frac{\log_5 z}{\log_5 5^{\frac{1}{3}}} = \frac{\log_5 z}{\frac{1}{3}} = 3 \cdot \log_5 z$

$$\rightarrow \underline{\underline{y = 3 \cdot \log_5(x+6) + 4}}$$

## Aufgabe 6



$$2r = s \cdot \sqrt{2}$$

$$s = r \cdot \sqrt{2}$$

$$2t = 2r - r \cdot \sqrt{2} = r \cdot (2 - \sqrt{2})$$

$$t = \frac{r \cdot (2 - \sqrt{2})}{2}$$

$$A = s^2 - \frac{s^2 \pi}{4} + 2 \cdot (t \cdot \sqrt{2})^2$$

$$= r^2 \cdot 2 - \frac{r^2 \cdot 2 \pi}{4} + 4 \cdot \frac{r^2 (2 - \sqrt{2})^2}{4}$$

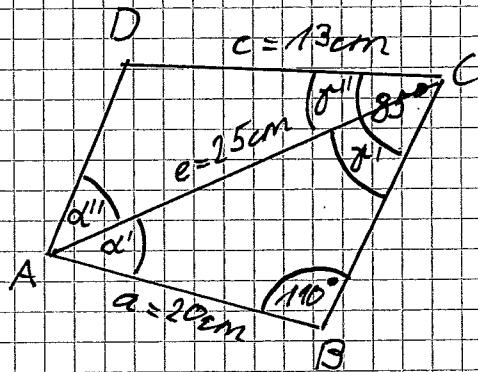
$$= r^2 \left( 2 - \frac{\pi}{2} + 4 - 4\sqrt{2} + 2 \right)$$

$$= r^2 \left( \frac{16 - \pi - 8\sqrt{2}}{2} \right)$$

$$= r^2 \cdot 0,7723$$



## Aufgabe 7



$$\frac{\sin \gamma'}{\sin 110^\circ} = \frac{20}{25} \longrightarrow \sin \gamma' = \sin 110^\circ \cdot \frac{20}{25} = 0,7578$$
$$\gamma'_1 = 48,74^\circ, \quad \gamma'_2 = 134,26^\circ$$

$$\alpha' = 180^\circ - \beta - \gamma'_1 = 180^\circ - 110^\circ - 48,74^\circ = 21,26^\circ$$

$$\frac{b}{e} = \frac{\sin \alpha'}{\sin 110^\circ} \longrightarrow b = 25 \cdot \frac{\sin 21,26^\circ}{\sin 110^\circ} = \underline{\underline{9,65 \text{ cm}}}$$

$$\gamma'' = \gamma - \gamma'_1 = 85^\circ - 48,74^\circ = 36,26^\circ$$

$$d = \sqrt{c^2 + e^2 - 2ce \cos \gamma''}$$
$$= \sqrt{13^2 + 25^2 - 2 \cdot 13 \cdot 25 \cdot \cos 36,26^\circ} = \underline{\underline{16,43 \text{ cm}}}$$

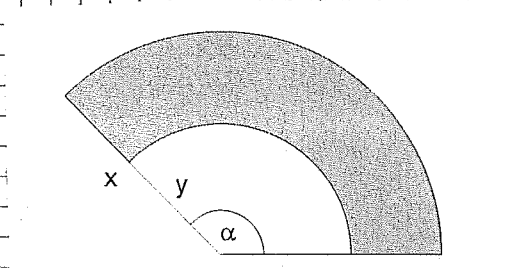
$$\frac{\sin \delta}{\sin \gamma''} = \frac{25}{d} \longrightarrow \sin \delta = \sin 36,26^\circ \cdot \frac{25}{16,43} = 0,90$$

$$\delta_1 = 64,16^\circ \longrightarrow \alpha_1'' = 79,56^\circ$$

$$\delta_2 = 115,84^\circ \longrightarrow \alpha_2'' = 27,91^\circ$$

$$\alpha = 360^\circ - \beta - \gamma - \delta = \underline{\underline{49,16^\circ}}$$

## Aufgabe 8



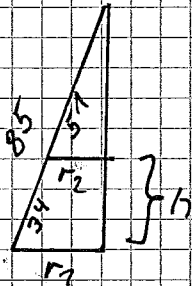
$$a) \quad 2r_1 \pi = \frac{2 \cdot 85 \cdot \pi \cdot 2880}{360 \cdot 17}$$

$$r_1 = \frac{85 \cdot 2880}{360 \cdot 17} = \underline{\underline{40 \text{ cm}}}$$

$$2r_2 \pi = \frac{2 \cdot 51 \cdot \pi \cdot 2880}{360 \cdot 17}$$

$$r_2 = \frac{51 \cdot 2880}{360 \cdot 17} = \underline{\underline{24 \text{ cm}}}$$

b)



$$h = \sqrt{34^2 - (r_1 - r_2)^2}$$

$$h = \sqrt{34^2 - 16^2}$$

$$h = \underline{\underline{30 \text{ cm}}}$$

c) Volumenanteil Spitze:

$$\left(\frac{51}{85}\right)^3 = \left(\frac{3}{5}\right)^3 = \frac{27}{125}$$

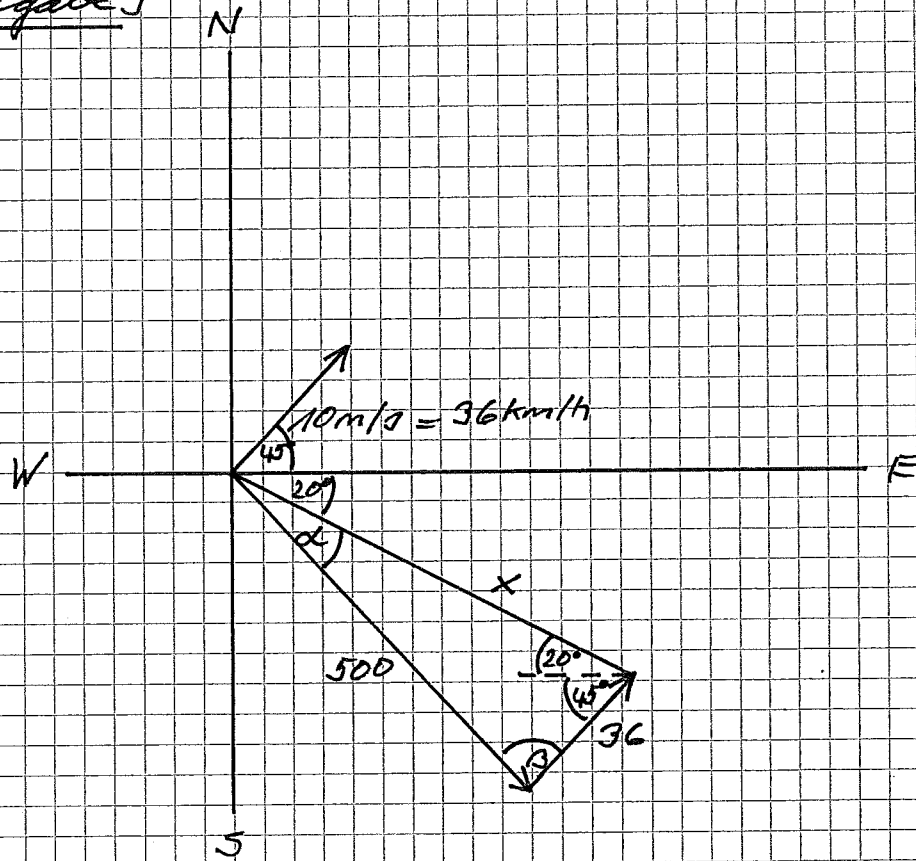
Volumenanteil Stumpf:

$$1 - \frac{27}{125} = \frac{98}{125}$$

Verhältnis: 98 : 125



# Aufgabe 9



a)

$$\frac{\sin \alpha}{\sin 65^\circ} = \frac{36}{500} \rightarrow \sin \alpha = \sin 65^\circ \cdot \frac{36}{500} = 0,0653$$
$$\alpha = \underline{\underline{3,74^\circ}}$$

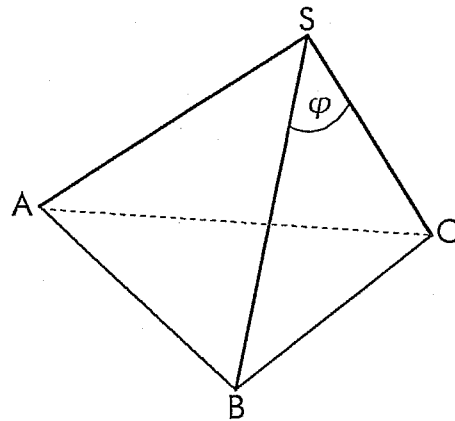
Steuerkurs: E 23,74^\circ S

b)  $\beta = 180^\circ - \alpha - 65^\circ = 111,26^\circ$

$$\frac{x}{500} = \frac{\sin \beta}{\sin 65^\circ} \rightarrow x = 500 \cdot \frac{\sin \beta}{\sin 65^\circ}$$

$$x = \underline{\underline{514,15 \text{ km/h}}}$$

# Aufgabe 10



$$\vec{SB} = \begin{pmatrix} 4-2 \\ 4-3 \\ 0-z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -z \end{pmatrix}$$

$$|\vec{SB}| = \sqrt{2^2 + 1^2 + z^2} = \sqrt{5+z^2}$$

$$\vec{SC} = \begin{pmatrix} 1-2 \\ 6-3 \\ 0-z \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -z \end{pmatrix}$$

$$|\vec{SC}| = \sqrt{(-1)^2 + 3^2 + z^2} = \sqrt{10+z^2}$$

$$\vec{SB} \cdot \vec{SC} = |\vec{SB}| \cdot |\vec{SC}| \cdot \cos \varphi$$

$$\begin{pmatrix} 2 \\ 1 \\ -z \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ -z \end{pmatrix} = \sqrt{5+z^2} \cdot \sqrt{10+z^2} \cdot \cos 60^\circ$$

$$-2 + 3 + z^2 = \sqrt{50 + 15z^2 + z^4} \cdot \frac{1}{2}$$

$$2 \cdot (1 + z^2) = \sqrt{50 + 15z^2 + z^4}$$

$$4(1 + 2z^2 + z^4) = 50 + 15z^2 + z^4$$

$$4 + 8z^2 + 4z^4 = 50 + 15z^2 + z^4$$

$$3z^4 - 7z^2 - 46 = 0$$

$$z_{1/2}^2 = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 3 \cdot (-46)}}{2 \cdot 3}$$

$$= \frac{7 \pm \sqrt{601}}{6} = \frac{7 \pm 24,52}{6}$$

$$z_1^2 = \frac{7 + 24,52}{6} = 5,253$$

$$\longrightarrow z_1 = \pm \sqrt{5,253}$$

$$\longrightarrow \underline{\underline{z = 2,292}}$$

$$z_2^2 = \frac{7 - 24,52}{6} = -2,919$$

$$\longrightarrow z_2 = \pm \sqrt{-2,919}$$